547 - Spring 2018 - HW4

February 14, 2018

1. Prove that if $X \xrightarrow{p} Y$ and $Y \xrightarrow{q} Z$ are covering spaces and the fibers of q are finite, then $X \xrightarrow{q \circ p} Z$ is a covering space.

2. Prove that if $X \xrightarrow{p} Y$ and $Y \xrightarrow{q} Z$ are such that q and $q \circ p$ are covering spaces, then so is p. In particular, this shows that every morphism in Cov_Z is itself a covering space.

3. Prove that a covering space $E \xrightarrow{p} B$ is regular if and only if $G := \mathrm{Aut}_{\mathrm{Cov}_B}(E \xrightarrow{p} B)$ acts transitively on $p^{-1}(b)$ for some (and hence every) basepoint b of B. Prove that in this case the orbit space $E/G \cong B$.

4. Fix a discrete group G. Recall that a G-regular covering space is a regular covering space $E \xrightarrow{p} B$ with $\mathrm{Aut}_{\mathrm{Cov}_B}(p) \cong G$. Suppose that there is no non-zero map $\pi_1(B, b) \to G$. Show that every G-regular cover of B is split, i.e., isomorphic to $B \times G$ as a cover.

5. Do Hatcher, Exercise 1.3.8.

6. Do Hatcher, Exercise 1.3.12.

7. Do Hatcher, Exercise 1.3.14.

8. Do Hatcher, Exercise 1.3.23.

9. What is the connection to Problem 3 from HW3? What seems to go wrong with that problem? Hint: consider the action of $\mathbb{Z}$ on S^1 obtained by rotating by an irrational angle. This is free. Is the quotient map a covering space map?