Problem Set 2
Math 446-3

Let $M = \Sigma^{-1}H^*(\mathbb{RP}^2, \mathbb{F}_2)$ as a module over the Steenrod algebra. This is a graded vector space of dimension 2, with a non-trivial $Sq^1$. It’s also the cohomology of the spectrum $\Sigma^{-1}\Sigma^\infty \mathbb{RP}^2 = M(\mathbb{Z}/2\mathbb{Z})$ – so named because it has a single non-vanishing integral homology group of $\mathbb{Z}/2\mathbb{Z}$ in degree 0.

1. Calculate $\text{Ext}^s(M, \Sigma^t \mathbb{F}_2)$ for $t - s \leq 4$ by using minimal resolutions.

2. Calculate $\text{Ext}^s(M, \Sigma^t \mathbb{F}_2)$ for $t - s \leq 10$ by using known results for $\text{Ext}^s(M, \Sigma^2 \mathbb{F}_2)$ and an appropriate long exact sequence.

3. Calculate $\pi_n(M(\mathbb{Z}/2\mathbb{Z}))$ for $n \leq 9$.

4. Calculate $\text{Ext}^t(M, \Sigma^t \mathbb{F}_2)$ for all $t$ and show that every element survives to $E_3$ is the Adams Spectral Sequence. The conjecture that they survive to $E_\infty$ is one of the great open problems in stable homotopy theory – it’s a strong form of the Kervaire invariant one question.