1. Complete the definition of the Thom spectrum $M(\xi)$ of a virtual bundle $\xi \in KO(X)$. In particular, that the construction given is well-defined. Also, show that if $f : X \to Y$ is continuous map and $\xi \in KO(Y)$, then there is an induced map of spectra
$$M(f^*\xi) \to M(\xi).$$

2. Let $\gamma$ be the canonical line bundle of $\mathbb{R}P^n$. Show that there is a homeomorphism
$$T(\gamma^k) \cong \mathbb{R}P^{n+k}/\mathbb{R}P^{k-1}.$$

3. Let $H_*$ denote ordinary homology, perhaps with coefficients. Define the homology of a spectrum $X$ by the formula
$$H_n(X) = \mathrm{colim} \hat{H}_{n+k}(X_k).$$

The purpose of this problem is define a Mayer-Vietoris sequence for homology. Define a morphism of spectra $X \to Y$ to be a cofibration if the induced map out of the push-out
$$S^1 \wedge Y_n \sqcup S^1 \wedge X_n \to Y_{n+1}$$
is a cofibration of spaces for all $n \geq -1$. Show that if $X \to Y$ is a cofibration and $f : X \to Z$ is any map of spectra, then there is a long exact sequence
$$\to H_n(X) \to H_n(Y) \oplus H_n(Z) \to H_n(Y \sqcup X Z) \to H_{n-1}(X) \to$$
which extends to infinity in both directions.

4. Suppose $X$ is a spectrum so that each space $X_n$ in the spectrum is $(n-1)$-connected and so that $H_k(X_n; k)$ is finitely generated in each degree, where $k$ is a finite field. Then we can define
$$H^k(X; k) = \lim H^{k+n}(X_n; k).$$
(In general there is an extra term.) Compute $H^*(HF_2; \mathbb{F}_2)$. You should get something familiar.

5. Prove that the model category structure on non-negatively graded chain complexes of $R$-modules described in class does, indeed, satisfy the axioms.