Math 320-1: Midterm 2

Northwestern University, Fall 2015

Name: ______________________________

1. (10 points) Give an example of each of the following. You do not have to justify your answer.

(a) A function on $\mathbb{R}$ which is nowhere continuous.

(b) A function $f : \mathbb{R} \to \mathbb{R}$ which is uniformly continuous on $[2, 100]$ but not on all of $\mathbb{R}$.

(c) A function on $\mathbb{R}$ which is differentiable but not twice differentiable.

(d) A function $f : \mathbb{R} \to \mathbb{R}$ which is differentiable at 3 and nowhere else.

Problem	Score
1	
2	
3	
4	
5	
Total	

2. (10 points) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a function such that $\lim_{x\to 2} f(x) = L$ exists and

$$2 < L < 5.$$

Show that there exists $\delta > 0$ such that $2 < f(x) < 5$ for all $x \in (2-\delta, 2+\delta)$ except possibly $x = 2$.

3. (10 points) Show that the function $f : (0, 4) \to \mathbb{R}$ defined by

$$f(x) = \frac{1}{x^2}$$

is continuous at $a = \frac{1}{3}$ and that it is not uniformly continuous on $(0, 4)$. When showing continuity at $\frac{1}{3}$ you MUST verify the ϵ-δ definition directly and cannot simply quote the fact that quotients of continuous functions are continuous whenever the denominator is nonzero.

4. (10 points) Suppose that $g : \mathbb{R} \to \mathbb{R}$ is continuously differentiable everywhere but not twice differentiable at 1. Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = (x - 1)g(x)$$

is twice differentiable at 1. Hint: The product rule will say right away that f is differentiable everywhere, but it won't immediately say that f is twice differentiable.

5. (10 points) Prove that $1 - \sin x \leq e^x$ for all $x \geq 0$. Hint: Find a good function to which you can apply the Mean Value Theorem.