Math 320-2: Midterm 1

Northwestern University, Winter 2020

Name: ______________________________

1. (10 points) Give an example of each of the following. You do not have to justify your answer.

(a) A sequence (a_n) for which $\sum a_n$ diverges but $\sum a_n^3$ converges.

(b) Continuous functions on $[-1, 0]$ which converge pointwise to a discontinuous function.

(c) A pointwise convergent series $\sum f_n(x)$ on $(-1, 1)$ such that $\sum f_n'(x)$ converges to $\frac{1}{1-x}$.

(d) A function which is not analytic on $(2, 3)$.

Problem	Score
1	
2	
3	
4	
5	
Total	

2. (10 points) Suppose (b_n) is a decreasing sequence of positive numbers which converges to 0. Show that the series $\sum_{n=0}^{\infty}(-1)^n b_n$ converges. Hint: How does the value of

$$b_n - b_{n+1} + b_{n+2} - b_{n+3} + \cdots + (-1)^k b_{n+k}$$

compare to the value of b_n?

3. (10 points) Determine, with justification, the value of ONE of the following limits:

$$\lim_{n\to\infty}\int_{-3}^{1} x^2 e^{x^2/n}\,dx \qquad \text{or} \qquad \lim_{n\to\infty}\int_{-3}^{1} (x^2+\sin^2(\tfrac{x}{n}))\,dx$$

You can use any inequality you've seen in class or on homework without justification.

4. (10 points) Show that the following series converges uniformly on any interval $[-M, M]$ centered at 0 in $\mathbb{R}$ and defines a differentiable function on all of $\mathbb{R}$.

$$\sum_{n=1}^{\infty} \left(1 - e^{x/n}\right)^2$$

You can take it for granted that for any $x \in \mathbb{R}$, $1 - e^{x/n} = \frac{x}{n} e^c$ for some c between 0 and $\frac{x}{n}$.

5. (10 points) Suppose $\sum_{n=0}^{\infty} a_n x^n$ has finite radius of convergence $R > 0$. Determine the largest open interval $(-L, L)$ centered at 0 on which the following series defines a differentiable function:

$$\sum_{n=0}^{\infty} 2^n a_n^2 x^{5n}$$