Math 320-2: Midterm 1
Northwestern University, Winter 2016

Name: ______________________________

1. (10 points) Give an example of each of the following. You do not have to justify your answer.

(a) A sequence (a_n) which converges to 0 but for which $\sum a_n$ diverges.

(b) A sequence of continuous functions on $[2, 3]$ which converges pointwise but not uniformly.

(c) A uniformly convergent series $\sum f_n(x)$ on $(-\frac{1}{2}, \frac{1}{2})$ such that $\sum f_n'(x)$ converges to $\frac{1}{(1-x)^2}$.

(d) A power series centered at 5 with radius of convergence $\frac{1}{3}$.

Problem	Score
1	
2	
3	
4	
5	
Total	

2. (10 points) Suppose (a_n) is a decreasing sequence of numbers for which $\sum_{n=1}^{\infty} a_n$ converges. Show that the sequence (na_{2n}) converges to 0. Hint: Use the fact that (a_n) is decreasing to bound $na_{2n} = \underbrace{a_{2n} + \cdots + a_{2n}}_{n \text{ times}}$.

3. (10 points) Determine the value of the following limit.

$$\lim_{n\to\infty} \int_0^4 \left(x^2 e^{x/n} - \frac{xn}{n+1} \right) \, dx$$

4. (10 points) Suppose for each $n \in \mathbb{N}$ the function $f_n : \mathbb{R} \to \mathbb{R}$ is differentiable and satisfies

$$|f_n(x)| \leq \frac{|x|}{n} \quad \text{and} \quad |f_n'(x)| \leq \frac{1 + \sin^2 x}{n} \text{ for all } x \in \mathbb{R}.$$

Show that $\sum_{n=1}^{\infty} f_n(x)^2$ converges pointwise to a differentiable function on $\mathbb{R}$.

5. (10 points) Determine the radius of convergence of the following series, and the explicit function to which it converges on its interval of convergence.

$$\sum_{k=1}^{\infty} k4^{2k}x^{4k}$$