Math 320-1: Midterm 1
Northwestern University, Fall 2019

Name: ______________________________

1. (10 points) Give an example of each of the following. You do not have to justify your answer.

(a) A subset of $\mathbb{R}$ with rational infimum and irrational supremum.

(b) A monotone sequence which does not converge.

(c) A Cauchy sequence whose terms are in the interval $(1, 5)$ but which does not converge to an element of this interval.

Problem	Score
1	
2	
3	
4	
5	
Total	

2. (10 points) Show that the supremum of the following set S is 3.

$$S = \left\{ \frac{3n+1}{n+\sqrt{n}} \;\middle|\; n \in \mathbb{N} \text{ and } n \geq 10 \right\}$$

3. (10 points) Suppose (x_n) is a sequence which converges to 2. Show, using the precise definition of convergence, that the sequence $(\frac{1}{x_n^2})$ converges to $\frac{1}{4}$. Hint: Figure out how to bound $|\frac{1}{x_n^2} - \frac{1}{4}|$ by a constant times $|x_n - 2|$, for large enough n.

4. (10 points) Suppose (x_n) is a convergent sequence. Show that the sequence (y_n) defined by

$$y_n = 4x_n + \frac{4\sin(n^2) - 3 + n^2\cos(n+1)}{4n^2 - n}$$

has a convergent subsequence.

5. (10 points) Suppose (x_n) is a sequence such that $x_n < 5$ for all $n \geq 100$. If (x_n) converges to x, show that $x \leq 5$. Hint: Show that $x > 5$ is not possible. (You cannot simply quote the "comparison theorem" from the book. The point is to give a proof of a special case of that theorem.)