1. Design a finite-state machine that inputs a string of a’s and b’s (read from left to right) and outputs the number of a’s plus twice the number of b’s modulo 4. For instance the input “abaababbaaba” would produce the output “1301302012312”.

2. Design a finite-state machine that inputs a string with an even number of binary digits (possibly padded with 0’s to the left) read from left to right and converts it into base 4. For instance it would translate the string “01001011” into “1023”.

3. In a computer system users must choose a password verifying the following requirements:
   (a) It must start with a letter.
   (b) It must end with a letter or digit.
   (c) It must contain at least a small letter, a capital letter and a digit.

In order to simplify the problem assume that the set of input symbols is \{s, c, d\} for “small letter”, “capital letter” and “digit” respectively. Design a finite-state automaton (as simple as possible) that accepts strings (of whatever length) verifying exactly the given criteria.

4. Design a deterministic finite-state automaton (by giving its transition diagram) that recognizes the strings represented by the regular expression $a(a + b)^*b$.

5. Let $G$ be the grammar with non terminal symbols \{E, T, F\}, terminal symbols \{a, +, *, (, )\}, start symbol $E$, and productions

   \[
   \begin{align*}
   E & \rightarrow T, & E & \rightarrow E + T, \\
   T & \rightarrow F, & T & \rightarrow T * F, \\
   F & \rightarrow (E), & F & \rightarrow a,
   \end{align*}
   \]

   (a) Find a derivation for the following string:

   \[a * a + (a + a * a)\]

   (b) Prove that the following string is not in the language $L(G)$ associated to the given grammar:

   \[a + a * (a + a * (a + a))\]

6. Let $G$ be the grammar with terminal symbols \{a, b\}, non terminal symbols \{S, T\}, start symbol $S$, and productions:

   \[
   \begin{align*}
   S & \rightarrow Sb, & S & \rightarrow Tb, & S & \rightarrow aT, & T & \rightarrow a
   \end{align*}
   \]

Prove that the language $L = L(G)$ associated to $G$ is regular by finding an equivalent grammar for $L$ that is regular.