ERRATUM TO "RESOLVENT ESTIMATES FOR NORMALLY HYPERBOLIC TRAPPED SETS", ANN. INST. HENRI POINCARÉ (A), 12(7)(2011), 1349-1385.

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In this erratum we correct three errors in the recent paper [3]. The errors are minor and do not affect the correctness of the principal results (although one mild hypothesis needs to be explicitly added). Descriptions of these errors and the necessary corrections are as follows. Note that this is the second revision of this erratum, now reflecting the addition of an explicit hypothesis that the trapped set should be symplectic.

- In §3.5 we omitted a crucial condition on G which is needed to have (3.24). In (3.20) we need to strengthen the second condition to

$$G = G_1 + \log(1/h) G_2, \quad \partial^\alpha H_p^k G_1 = \mathcal{O}((h/\tilde{h})^{-|\alpha|/2}), \quad k + |\alpha| \geq 1, \quad \partial^\alpha G_2 = \mathcal{O}(1).$$

  This is satisfied for the weight G in §§4.2–4.3. Expression (3.24) holds for $\ell \geq 2$, while the case $\ell = 1$ yields the slight variant:

$$\mathrm{ad}_{G^w(x,hD)} P \in h \log(1/h) \widetilde{\Psi}_{1/2}.$$

  The analysis follows from [5, §8.2] and is the same as in [2, §8]. See also [1, §7] and [4, Proposition 4.2] for similar arguments.

- Lemma 4.1 is incorrect as stated. The conclusion (4.4) does *not* hold for any defining functions of $\Gamma_\pm$ as can be seen by multiplying $\varphi_\pm$ by e^f and having $|H_f|$ large somewhere. *We are grateful to Semyon Dyatlov for pointing this out.*

  The error in the proof comes from the fact that C in the second displayed formula there may be greater than 1.

  The simple correction is to state that there exists *some* choice of defining functions satisfying (4.4) in some neighbourhood of K. We start with given defining functions $\tilde{\varphi}_\pm$ and then, similarly as in [2, Proof of Proposition 7.4] (but for defining functions rather than their squares as in [2]), set

$$\varphi_\pm(\rho) \stackrel{\text{def}}{=} \int_0^T \tilde{\varphi}_\pm(\exp tH_p(\rho))\, dt.$$

  These are defining functions of $\Gamma_\pm$ as these sets are invariant under the flow.

  Then

$$H_p \varphi_\pm(\rho) = \tilde{\varphi}_\pm(\exp TH_p(\rho)) - \tilde{\varphi}_\pm(\rho).$$

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Since $|\tilde\varphi_\pm(\rho)| \sim d(\rho, \Gamma_\pm)$, the second displayed formula in the proof of Lemma 4.1 with T large enough (for ρ in a T dependent neighbourhood of K), shows that

$$|\tilde\varphi_+(\exp TH_p(\rho))| \ll |\tilde\varphi_+(\rho)|, \quad |\tilde\varphi_-(\rho)| \ll |\tilde\varphi_-(\exp TH_p(\rho))|.$$

Hence

$$H_p\varphi_+(\rho) \sim -\tilde\varphi_+(\rho) \sim -\varphi_+(\rho), \quad H_p\varphi_-(\rho) \sim \tilde\varphi_-(\exp TH_p(\rho)) \sim \varphi_-(\rho),$$

with constants depending on T. This gives (4.4).

- The assertion, in Dynamical Hypothesis (2), that K must automatically be symplectic, seems to be false. *We must therefore add the hypothesis that K is symplectic,* as this fact is used crucially in the end of the proof of Lemma 4.1, where we observe that $\{\varphi_+, \varphi_-\} \neq 0$. *We are grateful to Semyon Dyatlov for pointing this out.*

References

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