

MATH 465-1, FALL 2012: A MENAGERIE OF MANIFOLDS AND CLASSIFYING SPACES

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This class will form a hands-on tour of some of the most interesting spaces in mathematics, with a focus on computing cohomology rings, homotopy groups, and characteristic classes. We will develop and use the theory of spectral sequences for these computations; we will also incorporate some of rational homotopy theory, Morse theory, and the Steenrod algebra, depending on participants' interests. Highlights will include Bott's periodicity theorem and Quillen's computation of the algebraic K-theory of finite fields. Spaces to be studied in particular detail will include:

- Lie groups. Grassmannians, flag varieties, and homogeneous spaces.
- Hypersurfaces in complex projective spaces. Complex algebraic varieties.
- Configuration spaces $\text{Conf}_i(M)$ and their compactifications.
- Classifying spaces $\text{BGL}_n(\mathbb{F}_q)$ of general linear groups over a finite field. Classifying spaces $\text{B}\Sigma_n$ of symmetric groups.

Additional spaces, based on participants' interests, may be drawn from the theory of 3-manifolds, hyperbolic manifolds, simply connected 4-manifolds, diffeomorphism groups of surfaces, exotic spheres, and mapping spaces.

Prerequisites: Basic knowledge of algebraic topology, manifolds, vector bundles, cohomology, and characteristic classes.