Homework 2, due 2/13

1. Which of the following pairs of topological spaces are homeomorphic? Justify your answer.

   (a) $X = [0, 1]$ and $Y = (0, 1)$,

   (b) $X = (0, 1)$ and $Y = \mathbf{R}$,

   (c) $X = \mathbf{Z}$ and $Y = \{0\} \cup \{1/n : n = 1, 2, 3, \ldots\}$, as subspaces of $\mathbf{R}$.

2. Let X be a topological space and $A \subset X$ a subspace. Show that the inclusion map $j : A \to X$ is continuous. *(This is Theorem 18.2 in the book, but write it up in your own words.)*

3. (a) Show that the metric topology on every metric space X has the following property: if $x, y \in X$ are distinct, then there are disjoint open sets $U, V \subset X$ such that $x \in U$ and $y \in V$. (This is called the Hausdorff property.)

   (b) Show that the topology $\mathcal{T}$ on $\mathbf{R}$ defined in Question 1 on Homework set 1 is not induced by any metric on $\mathbf{R}$.

4. Let $S^1 \subset \mathbf{R}^2$ be the unit circle $x^2 + y^2 = 1$. Let $f : S^1 \to \mathbf{R}$ be continuous. Show that there is a point $x \in S^1$ such that $f(x) = f(-x)$. *(Hint: use the intermediate value theorem.)*