

Putnam Questions, Week 1

1. Prove that $\frac{1}{1 + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \dots + \frac{1}{\sqrt{99} + \sqrt{100}} = 9$.
2. Prove that there exists an integer n such that the first four digits of 2^n are 2, 0, 0, 9.
3. Given a set of $n + 1$ integers between 1 and $2n$, prove that one number must divide another. Prove that this is not necessarily true for n integers between 1 and $2n$.
4. Let $f(x)$ be a polynomial, and suppose that $f(x) + f'(x) > 0$ for all x . Prove that $f(x) > 0$ for all x .
5. For which real numbers c is $(e^x + e^{-x})/2 \leq e^{cx^2}$ for all real x ?
6. Evaluate the infinite product $\prod_{n=2}^{\infty} \frac{n^3 - 1}{n^3 + 1}$.