Math 215

Final Exam

Fall 2001

Name: ______________________

I.D. # ______________________

Instructions: Write your name and I.D. number above. No books, calculators, notes or tables are allowed. You must show all work on these pages, and make sure that your final answer is clearly shown.

Circle the name and section of your instructor:

K. Vilonen	12:00
J. Mauger	10:00
J. Mauger	12:00
M. Kang	1:00

Prob.	Full Points	Score
1	15	
2	25	
3	35	
4	25	
5	30	
6	35	
7	35	
TOTAL	200	

In this exam, $\oint_C \mathbf{F} \cdot d\mathbf{r}$ and $\oint_C \mathbf{F} \cdot \mathbf{T}\, ds$ have the same meaning.

Some formulas:

$$\int_0^{2\pi} \sin^n t\, dt = \int_0^{2\pi} \cos^n t\, dt = \begin{cases} 0 & \text{if } n \text{ is odd,} \\ \dfrac{1 \cdot 3 \cdot 5 \cdots (n-1)}{2 \cdot 4 \cdot 6 \cdots n} 2\pi & \text{if } n \text{ is even.} \end{cases}$$

1. By changing the order of integration, evaluate the following iterated integral

$$\int_0^1 \int_{\sqrt{x}}^1 \frac{5x}{1+y^5} \; dy\, dx$$

2. Let $\mathbf{F} = \langle\, y^2 \,,\, 2xy - z\sin(yz) \,,\, -y\sin(yz) + 6z^2 \,\rangle$.

(a) Find a potential function for $\mathbf{F}$.

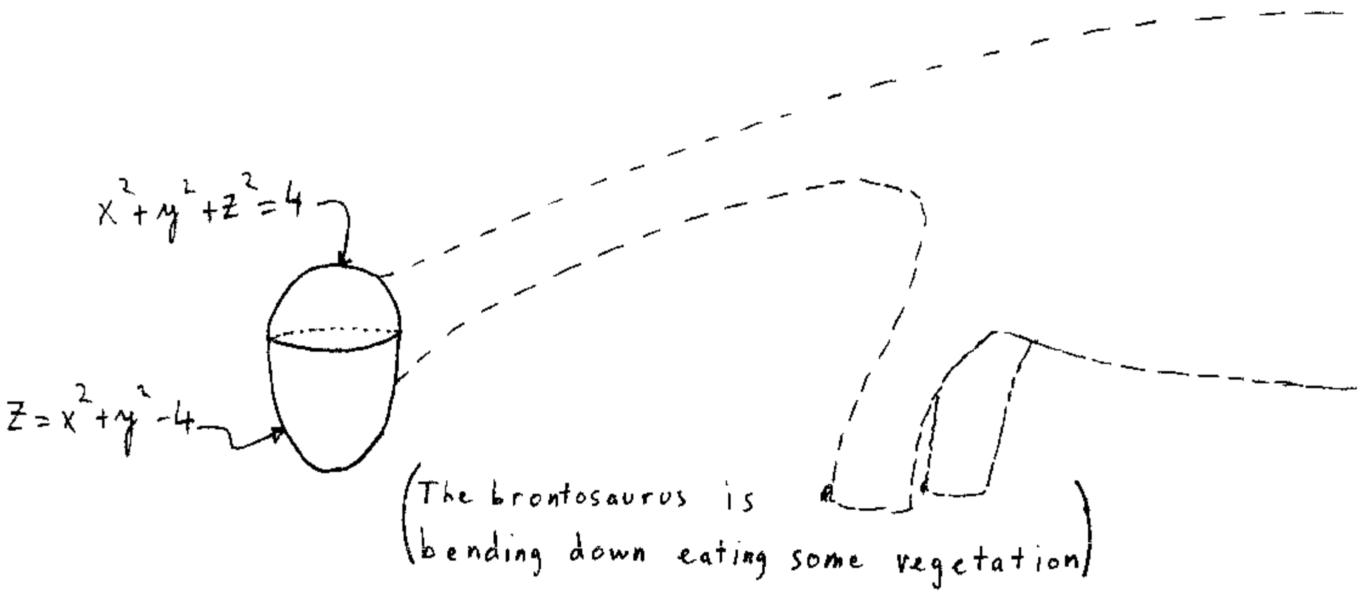
(b) Find the work done by $\mathbf{F}$ in moving a particle along the curve $\mathbf{C}$ parametrized by

$$\mathbf{r}(t) = \langle\, t^4 \,,\, te^{t^2} - e^{t^3}\sin\left(\frac{\pi}{2}t\right) \,,\, 1 - \cos\left(\frac{\pi}{2}t\right) \,\rangle,$$

where t runs from 0 to 1.

3. A brontosaurus' head is (roughly) the solid region bounded by the part of the paraboloid $z = x^2 + y^2 - 4$ for $z \leq 0$ and by the hemisphere $x^2 + y^2 + z^2 = 4$ for $z \geq 0$ [refer to picture].

(**a**) Find the volume of the brontosaurus' head.



Let **S** denote the surface of the brontosaurus' head oriented with the outward normal vector. Let **F** be the vector field $\mathbf{F} = y^2\,\mathbf{i} + yz\,\mathbf{j}$ (the **k** component of **F** is zero).

(b) Use the divergence theorem to compute the flux of **F** through **S**.

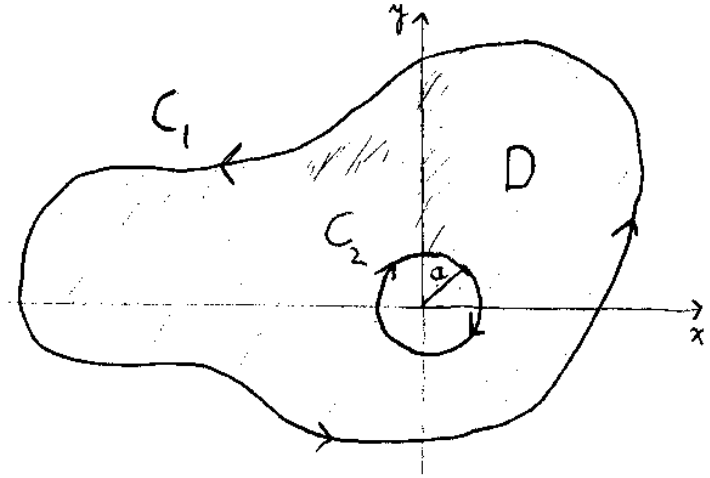
(c) Let $\mathbf{S}_1$ be the lower part of $\mathbf{S}$ that lies in the region $z \leq 0$ with the outward normal vector. Compute $\iint_{\mathbf{S}_1} (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$. You might try using Stokes' theorem.

4. By using an appropriate change of variables, evaluate the following integral:

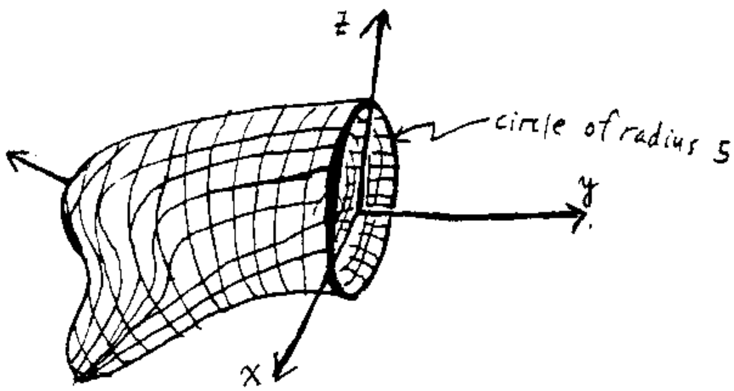
$$\iint_D e^{\frac{x^2}{4}+\frac{y^2}{9}} \; dy\, dx,$$

where D is the region given by $\frac{x^2}{4}+\frac{y^2}{9} \leq 1$. One way to do it would be to rescale x and y by some appropriate constants, and then use polar coordinates.

5. Let $\mathbf{F} = \langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \rangle$. Let C_1 be the outer curve in the picture, and let C_2 be the inner circle of radius a centered at the origin. Orient C_1 counterclockwise and C_2 clockwise. Find the work done by $\mathbf{F}$ along the curve $\mathbf{C}_1$ by applying Green's theorem to the shaded region D between $\mathbf{C}_1$ and $\mathbf{C}_2$.



6. A fisherman fixes the rim of his net in the sea. The rim is a circle of radius 5 in the xz plane centered at the origin. The rest of the net consists of a mesh bag whose edge is the circular rim (see the figure below).



You have *no* knowledge of the position of the mesh part of the net, but you do know the velocity of the water is given by the vector field

$$\mathbf{F} = (x^4 + 2y^2)\mathbf{i} + (-3 - y^2)\mathbf{j} + (2yz - 4x^3z)\mathbf{k}$$

Compute the flux of the water (i.e. the flux of the vector field $\mathbf{F}$) through the mesh with the indicated normal vector.

7. A broken wine bottle sits on the xy plane as shown. It consists of a portion of a cylinder of radius 1 along the z axis, with the unit disc in the xy plane at the bottom (see the figure below). Let **C** be the path along the broken edge oriented counterclockwise when viewed from above. If **F** is the vector field $\mathbf{F} = \langle -y , 2x , 10z \rangle$, what is the value of $\oint_C \mathbf{F} \cdot d\mathbf{r}$?