Math B21 Final December 12, 1997 C. Robinson

1. (24 Points) Multiple Choice: For each of the equations in column I, pick the form of the solution from column II that best matches it. (You may use the choice from column II more than once.)

Column I: Equations	Column II: Form of the solution
(a) $y'' + 4\,y = \cos(x/2)$	(i) e^{rx}
(b) $y'' + 4\,y = \cos(2x)$	(ii) $A\cos(\omega x) + B\sin(\omega x)$, for appropriate value of ω
(c) $y'' + 5\,y' + 4\,y = \cos(2x)$	(iii) $Ax\cos(\omega x) + Bx\sin(\omega x)$, for appropriate value of ω
(d) $x^2\,y'' + y' + (3 - 2\,x + x^2)\,y = 0$	(iv) $\sum_{n=0}^{\infty} a_n\,x^n$
(e) $x^2\,y'' + x\,y' + (3 - 2\,x + x^2)\,y = 0$	(v) $\sum_{n=0}^{\infty} a_n\,x^{n+r}$
(f) $(4 - x^2)\,y'' + x\,y' + (3 + x^2)\,y = 0$	(vi) None of the above

2. (26 Points) (a) Find the general solution of $y' + 2\,x\,y = 2\,x$.
   (b) Find the solution with $y(0) = 3$.

3. (25 Points) Using the **improved Euler method** with step size $h = 0.1$, find an approximate value of the solution at $t = 0.2$ for the initial value problem

$$\frac{dy}{dt} = 2\,y + t \qquad y(0) = 1.$$

4. (25 Points) Find the general solution of the linear system of differential equations

$$\dot{\mathbf{x}} = \begin{pmatrix} 2 & 1 & 0 \\ -5 & 0 & 1 \\ 0 & 0 & 3 \end{pmatrix} \mathbf{x}.$$

   Note: the eigenvalues are 3 and $1 \pm 2i$

5. (25 Points) The indicial equation for the differential equation

$$2\,x^2\,y'' + (x^2 - 3\,x)\,y' - 3\,y = 0$$

   is $2r^2 - 5r - 3 = 0$. For each of the two independent Frobenius solutions, is it defined at $x = 0$ or does it have a vertical asymptote? You do not need to find the coefficients for the two solutions.

6. (25 Points) Consider the series solution of the differential equation

$$y'' - x\,y' - 2\,y = 0$$

   in terms of powers of x.
   (a) Find the recurrence relation for the coefficients.
   (b) Find the first three terms of each of the two independent solutions.

7. (25 Points) Consider the differential equation

$$y'' - 6\,y' + 9\,y = 0.$$

   (a) Find the general solution.
   (b) Find the maximal interval containing $x = 1$ on which these two solutions form a fundamental set of solutions (are independent). Justify your answer.

8. (25 Points) Consider the system of differential equations

$$\dot{x} = 1 - y^2$$
$$\dot{y} = x - 2\,y.$$

   (a) Find the equilibria (critical points).
   (b) Indicate the stability type of each equilibria (asymptotically stable, stable, unstable, or undetermined by the linearization).