NAME:_____ Score: _____ out of 200

Math B14-3, Section 31 March 19, 1997

Final Exam

Be sure to explain each answer clearly. This means that for any answer which requires several steps (as will happen a few times on this exam), do not just write down a few formulas, numbers, or vectors, followed by the answer. Tell what you're doing.

Question 1. (25 points.) Define each of the following terms clearly and concisely:

(a) limit of a function $f(x, y)$ as $(x, y) \to (a, b)$

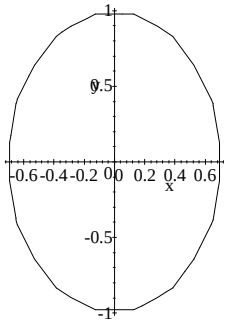
(b) trace (of a surface)

(c) dot product (of two vectors, each with three components)

(d) speed (of a particle moving along a parametric curve)

(e) gradient (of a function $f(x, y, z)$)

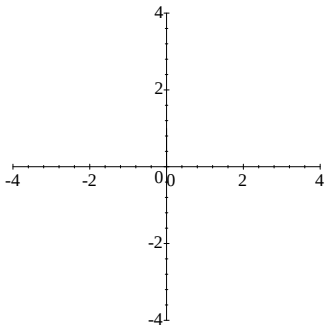
Question 2. (25 points.) Using any methods from the class, find the absolute maximum of the function $f(x, y) = x + y$ on the region bounded by the ellipse $2x^2 + y^2 = 1$ (that is, the region which includes the ellipse and its interior). Explain how you are solving the problem, i.e., why your answer is really an absolute maximum for the function on the whole region. Mark the point in the region where f takes its maximum. Note that the value of f at a point in the plane is the sum of the x and y coordinates, so point you have found should have the largest sum of x and y coordinates among all the points in the region.



Question 3. (15 points.) Graph the curve given by the parametric equations

$$\begin{aligned} x(t) &= 2 - 2\cos t \\ y(t) &= 1 + \sin t \end{aligned}$$

for $0 \leq t \leq 2\pi$. Find the value of $\frac{dy}{dx}$ at the point on the curve where $t = \frac{\pi}{4}$. Mark this point on the graph and draw the tangent line to the curve at this point. (Hint: make sure the slope of the tangent line looks like it is close to what you know it should be.) Eliminate the parameter to find an equation involving just x and y whose graph is this curve.

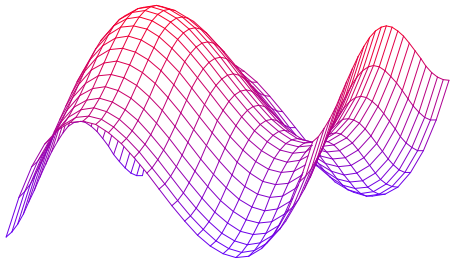


Question 4. (15 points.) Use differentials to estimate $\sqrt{\sqrt{25.1} - \sqrt{1.1}}$ (this will actually give you an answer you can find without a calculator which is accurate to three decimal places).

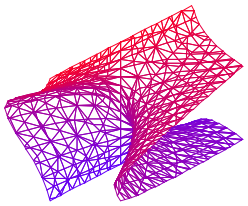
Question 5. (20 points.) Find and classify the critical points of the function

$$f(x, y) = 37 \cos x + y^3 - 9y^2 + 1$$

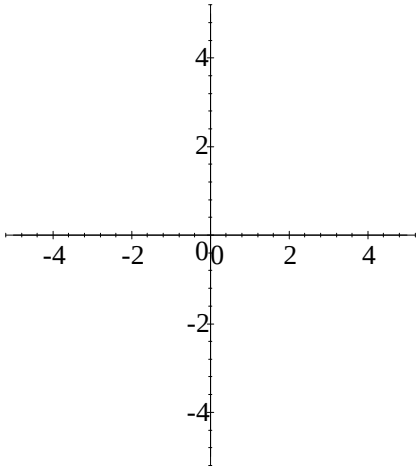
in the region where $-3 < x < 3$ and $-3 < y < 9$, and mark the points on the graph below.



Question 6. (15 points.) Find the equation of the tangent plane to the graph, shown below, of the equation $z^2y+4x-y^2=0$ at the point $(0,1,1)$.

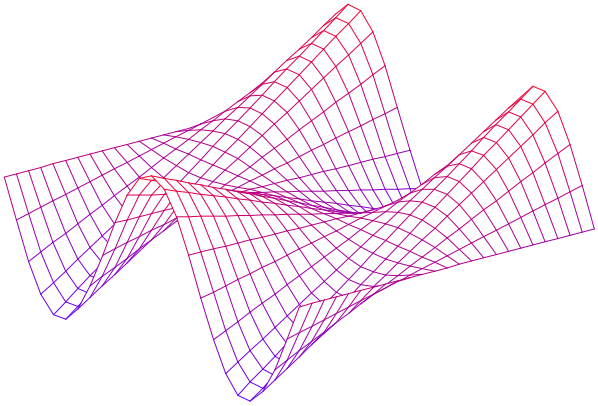


Question 7. (15 points.) Draw the level curves (for at least three different levels) of the function $f(x,y)=x^2+y+2x$ and label each curve with its level.

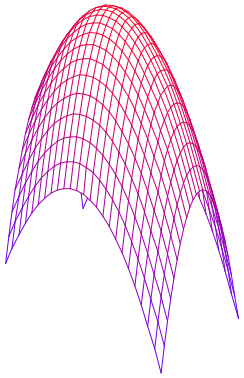


Question 8. (20 points.) Each of the pictures below is the graph of one of the functions listed, as usual, with the axes in their normal positions (as in class and in the book). Identify each. You do not need to show your work here.

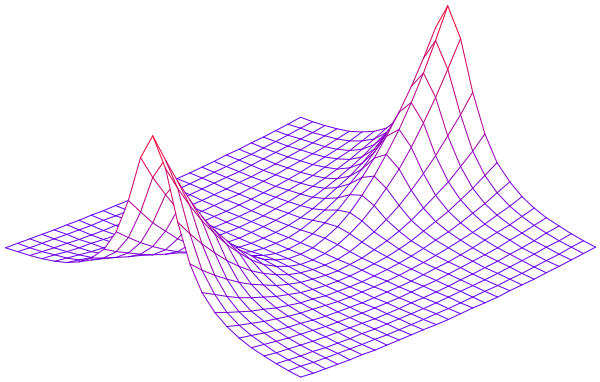
a) $f(x,y) = \frac{x^2}{1+y^2}$
b) $f(x,y) = x \sin y$
c) $f(x,y) = \frac{1}{1+x^2+y^2}$
d) $f(x,y) = x - \frac{1}{5}y^2$
e) $f(x,y) = -2x^2 - \frac{2}{5}y^2$
f) $f(x,y) = 4xy$



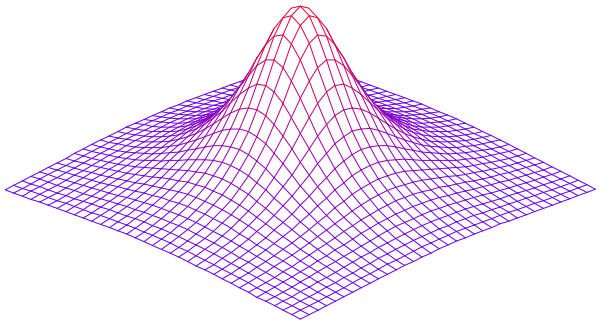
$z =$ _____



$z =$ _____

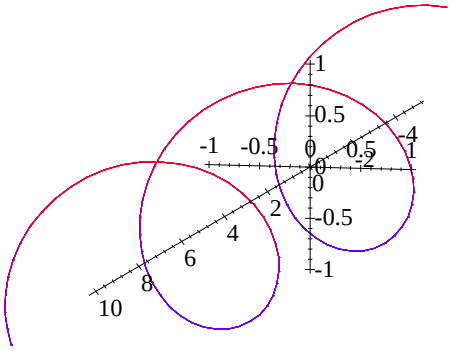


$z =$ _____



$z =$ _____

Question 9. (20 points.) A particle moves along a helical path with position vector $\mathbf{r}(t) = \langle t, \cos t, \sin t \rangle$. Its path is shown below. Find the velocity, speed, and acceleration at time t, and find the unit tangent vector $\mathbf{T}(t)$, unit normal vector $\mathbf{N}(t)$, and curvature $\kappa(t)$. Draw the unit tangent and normal vectors on the diagram at the point of the curve where $t = 2\pi$.



Question 10. (15 points.) Let $f(x, y, z) = y + \frac{x}{1+z^2}$.

(a) Find $D_{\langle 0, \frac{3}{5}, \frac{4}{5} \rangle} f(1, 1, 1)$.

(b) Further let x, y, and z be functions of u and v, as follows:

$$\begin{aligned} x &= u + v \\ y &= uv^2 \\ z &= u \end{aligned}$$

Find $\frac{\partial f}{\partial u}$ expressed as a function of u and v.

Question 11. (15 points.) Express the vector $\mathbf{a} = \langle 3, 1, 0 \rangle$ as the sum of a vector ($\mathbf{a}_\parallel$) parallel to the vector $\mathbf{b} = \langle 1, 0, 1 \rangle$ and a vector ($\mathbf{a}_\perp$) perpendicular to $\mathbf{b}$. What is $\mathbf{a}_\perp \times \mathbf{a}_\parallel$? What is $\mathbf{a}_\perp \cdot \mathbf{a}_\parallel$?