

Final Exam

I.D. # \_\_\_\_\_

Winter 1997

**Instructions:** Write your name and I.D. number above. Show all work on these pages, and make sure that your final answer is clearly shown. No **books**, **calculators**, or **tables** are allowed. Check that this exam contains pages 1–6. Good luck, and have a nice holiday!

**Circle the name of your instructor:**

DiBenedetto	8:00
DiBenedetto	9:00
GChristofi	10:00
Abdulali	10:00
Silber	11:00
Welland	12:00
Volpert	1:00

### Some Useful Formulas

Area of a circle	$A = \pi r^2$	Surface Area of a sphere	$A = 4\pi r^2$
Circum. of a circle	$p = 2\pi r$	Volume of a cone	$V = \frac{1}{3} \pi r^2 h$
Area of a trapezoid	$A = \frac{1}{2} h(b_1 + b_2)$	Volume of a cylinder	$V = \pi r^2 h$
Volume of a sphere	$V = \frac{4}{3} \pi r^3$	Lateral surface area of a cylinder	$A = 2\pi r h$

Prob.	Possible points	Score
1	35	
2	21	
3	10	
4	10	
5	20	
6	20	
7	17	
8	17	
9	15	
10	35	
<b>TOTAL</b>	200	

1. Compute each of the following derivatives. Do not simplify.

(a)  $\frac{d}{dx}(7x^6 - 5x^{-3} + \frac{1}{\sqrt{x}})$

(b)  $\frac{d}{dt}(t^3 - 3)(t^{\frac{3}{2}} - 2t^{-5})$

(c)  $\frac{d}{dx} \frac{3x^2 - 1}{2x + 3}$

(d)  $\frac{d}{ds}(\sin(3s) + s)^5$

(e)  $\frac{d}{dx} \int_2^{2x} (t^2 + 1) dt$

2. Evaluate the following integrals:

(a) 
$$\int_2^3 \frac{t^{\frac{3}{2}} + t^{\frac{1}{2}} + 1}{t^3} dt$$

(b) 
$$\int \frac{1}{\sqrt{x}(1 + \sqrt{x})^3} dx$$

(c) 
$$\int_0^{\frac{\pi}{2}} \sin 2x \cos 2x dx$$

3. At which of the five indicated points on the graph sketched below do  $y'$  and  $y''$  have the same sign? Explain your answer.

=5trueinprob3.eps

4. Below is the graph of  $f(x)$ . Sketch the graph of  $f'(x)$ .

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=5trueinaxes.eps

5. A rocket that is launched vertically is tracked by a radar station located on the ground 4 miles from the launch site. What is the vertical speed of the rocket at the instant its distance from the radar station is 5 miles and this distance is increasing at the rate of 3600 mi/hr?

6. Find the point or points on the graph of  $y = x^2 + 2$  closest to the point  $(0, 4)$ .
7. Use implicit differentiation to find  $y'$  for the function  $y = y(x)$  satisfying the equation  $x^3 + 3x^2y + y^2 = 9$ . Find the equation of the tangent line to the graph of this equation at the point  $(1, 2)$ .

8. Find the area above the graph of  $y = x^2 - 4x$  and below that of  $y = x$ .

9. Use Riemann sums to obtain an estimate for

$$\int_1^3 x^2 dx$$

Divide the interval  $[1, 3]$  into 4 equal subintervals  $[x_{i-1}, x_i]$ . Use  $x_i^* = x_i$  the right hand endpoint of the  $i$ -th subinterval  $[x_{i-1}, x_i]$ .

10. Consider  $f(x) = \frac{x}{x^2 - 1}$ ,  $f'(x) = -\frac{x^2 + 1}{(x^2 - 1)^2}$ ,  $f''(x) = \frac{2x(x^2 + 3)}{(x^2 - 1)^3}$ . Find:

- (a) the maxima and the minima,
- (b) the vertical and horizontal asymptotes,
- (c) the intervals where the function is concave upward, those where it is concave downward and the inflection points, and
- (d) sketch the graph of  $f$ .