B14-1 Final Exam Partial Answers **December 7, 1995**

Answers are not guaranteed. If in doubt, consult your instructor or TA.

1.

(a) $5x^4 + x^{-\frac{4}{3}} + 20x^{19}$

(b) $(3(x^2+5)^2(2x)(x^3-6x)^6 + (x^2+5)^3 6(x^3-6x)^5(3x^2-6)$

(c) $\frac{1}{2}\sqrt{\frac{x^2+9}{3x+1}}\frac{3(x^2+9)-(3x+1)(2x)}{(x^2+9)^2}$

(d) $\sec(1+x^2)\tan(1+x^2)(2x) - \csc^2(x+\sin x)(1+\cos x)$

(e) $\frac{1}{1+x^6}(3x^2)$

2.

(a) $\frac{4}{3}$

(b) $\frac{(x^2+6)^{21}}{42} + C$

(c) $\frac{(3+\sin x)^5}{5} + C$

3. The terminolgy is not accurate. The question should have asked for 'local' mimima and 'local' maxima.

$(-2,-32)$ is a local minimum, $y'' = 72$. $(0,0)$ is a local maximum, $y'' = -24$. $(1,-5)$ is a local minimum, $y'' = 36$.

4. $y - 2 = 2(x-1)$ or $y = 2x$

5. $.36\pi$. Tricky point: $dh = .04$ since both top and bottom contribute to change in height.

6. $15/26 = 0.577$ yards per sec.

7. Side of base, $x = 3$, height, $y = 6$. As a function of x, the cost is given by $4x^2 + \frac{216}{x}$ for $0 < x < \infty$. The second derivative at $x = 3$ is positive so this is a local minimum. *Since there is a unique critical point* and the function is everywhere differentiable, this suffices for us to conclude that the local minimum must be the absolute minimum. Alternately, the first derivative is negative for $0 < x < 3$ and positive for $3 < x < \infty$. This allows us to visualize the graph of the function and conclude the critical point at $x = 3$ is an absolute minimum.

8. $x_1 = \frac{1}{2}, x_2 = 3$.

9. $\frac{2}{3}$. The line intersects the graph of $y = x^5$ in three points, so the integral breaks into two part. By symmetry, however, the two areas are equal, so you can find one and double it.

10.

(a) Vertical: $x = -5, x = 5$; horizontal $y = 1$.

(b) Increasing: $(-\infty,-5), (-5,0)$; decreasing: $(0,5), (5,\infty)$.

(c) Concave upwards: $(-\infty,-5), (5,\infty)$; concave downwards: $(-5,5)$; no inflection points,

(d) There is one critical point at $(0,0)$ which is a local maximum.

Use Maple to check if your hand drawn graph is correct.

11. $\frac{f(x+h)-f(x)}{h} = \cdots = \frac{x+4-(x+4+h)}{h(x+4)(x+4+h)} = -\frac{1}{(x+4)(x+4+h)}$. Now take the limit as $h \to 0$ to get $-\frac{1}{(x+4)^2}$