

FINAL EXAMINATION

MATH A10

December 8, 1999

Solve the following problems. Show all work, be neat. The maximum score for this test is 200 points. Answers like $C_{52,3}$ or 3^{12} are fine, unless you are asked to find an answer numerically. Good luck !

1. (10 points) Consider the universal set $\mathcal{U} = \{a, b, c, d, e, f, g, h\}$ and let $A = \{a, b, c, f, g, h\}$, $B = \{c, d, e, f\}$, $C = \{b, g\}$. Find
 - a) $(C \cup B) \cap A$
 - b) A'
 - c) $(C \cap B)'$.

2. (10 points) One hundred people were polled on which web browser do they use on their personal computer. It turned out that 45 use Netscape; 40 use Explorer; 41 use AOL; 15 use Netscape and AOL; 11 use Netscape and Explorer; 9 use Explorer and AOL; 5 use all three.
 - a) How many of the people polled do not use any of these three browsers?
 - b) How many use Netscape and Explorer but not AOL?

3. (10 points) Using the binomial formula write in expanded form with **numerical** coefficients $(2x + y)^5$.

4. (10 points) Evaluate **numerically** the expressions:
 - a) $P_{7,2}$
 - b) $C_{9,3}$
 - c) $\frac{5!+4!}{(5+4)!}$.

5. (20 points) A standard deck has 52 cards.
 - a) Suppose you put them one by one on a long table from left to right. In how many ways can you do that?
 - b) How many are the 4 card hands containing 1 jack, 1 queen, 1 king and 1 ace?
 - c) How many are all possible 10 card hands?
 - d) How many are the the 13 card hands containing 3 clubs, 3 diamonds, 3 hearts and 4 spades?

6. (10 points) How many different rearrangements are there of the letters of the word CINCINNATI.

7. (20 points) An urn contains 3 white, 4 blue and 5 red balls. Select 4 of them without looking inside.
 - a) What is the probability that all 4 are blue?
 - b) What is the probability that all 4 are red?
 - c) What is the probability that you 2 are white, 1 is blue and 1 is red?

8. (10 points) Let A and B be two events such that $P(A) = 0.3$, $P(A \cup B) = 0.5$ and $P(B) = p$ (unknown).
 - a) If A and B are mutually exclusive events, find the value of p .
 - b) If A and B are independent events, find the value of p .

[Hint: Use the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.] *(please, turn over)*

9. (10 points) Suppose you have a random variable X with values and corresponding probabilities given in the following table

X	-2	-1	0	1	2
p	0.1	0.1	0.1	0.1	0.6

Find **numerically**

- the expected value $E(X)$
 - the variance $Var(X)$.
10. (20 points) A businesswoman has two investment possibilities: project A and project B. If she invests in project A, there is a 30% chance she will lose \$10,000, a 50% chance she will break even and a 20% chance she will make \$15,000. If she invests in project B, there is a 15% chance she will lose \$20,000, a 40% chance she will break even and a 45% chance she will make \$8,000. Determine the expected profit for each project and decide in which project should she invest?
11. (10 points) Pete commutes every day to work from Evanston to downtown Chicago by car. Over the last year he has found out that the expected length of the trip is 55 min with a standard deviation of 5 min. Using Chebyshev's inequality estimate the probability that on a given day the trip is going to last anywhere between 40 min and 70 min.
12. (20 points) Carrie takes a multiple-choice test in a foreign language she does not know. The test has 30 questions and each question has 5 answers only one of which is correct. Do the following, leaving your answers in combinatorial formulas.
- For a given question find the probability that she guesses the right answer.
 - Find the probability that she guesses the correct answer for exactly 15 questions.
 - If getting an A means having the right answer for at least 27 questions, find the probability that she gets an A on the test.
13. (10 points) Let X be a standard normal variable ($\mu = 0$, $\sigma = 1$) and let Y be a normal variable with $\mu = 10$ and $\sigma = 2$. Use the normal distribution tables to do the following:
- Find $P(-0.31 \leq X \leq -0.06)$.
 - Find $P(Y \geq 11.6)$.
14. (10 points) Let X be a binomial random variable which is the number of successes in 200 Bernoulli trials, (i.e. $n = 200$) with a probability of success in every individual trial being $p = 0.05$.
- Find the mean and standard deviation of X .
 - Using normal approximation find **numerically** the probability $P(5 \leq X \leq 20)$.
15. (20 points) Years ago in a snowy January in Evanston the following snowfalls (in inches) were recorded in the course of 10 days: 2, 4, 5, 2, 6, 0, 2, 4, 5, 8.
- Draw a histogram for this observation.
 - Write the expression for the sample mean and compute it **numerically**.
 - Find the median and the mode.
 - Write the expression for the sample variance, and compute it **numerically**.