

Math 221-0
Final Exam

Name: _____

To receive full credit, you must show all of your work. The use of calculators is not permitted.

Problem	Points	
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	
Total	200	

1. (20 points) Find the general solution of

$$y' = \sqrt{t(y+1)}$$

Answer: _____

2. (20 points) Find the general solution of

$$(\cos y - te^y)y' + 2t - e^y = 0$$

Answer: _____

3. (10 points each) Consider the initial value problem

$$\frac{dy}{dt} = 2ty + 4t, \quad y(0) = -1$$

a) Use Euler's method with $n = 2$ steps to approximate $y(1)$.

Answer: _____

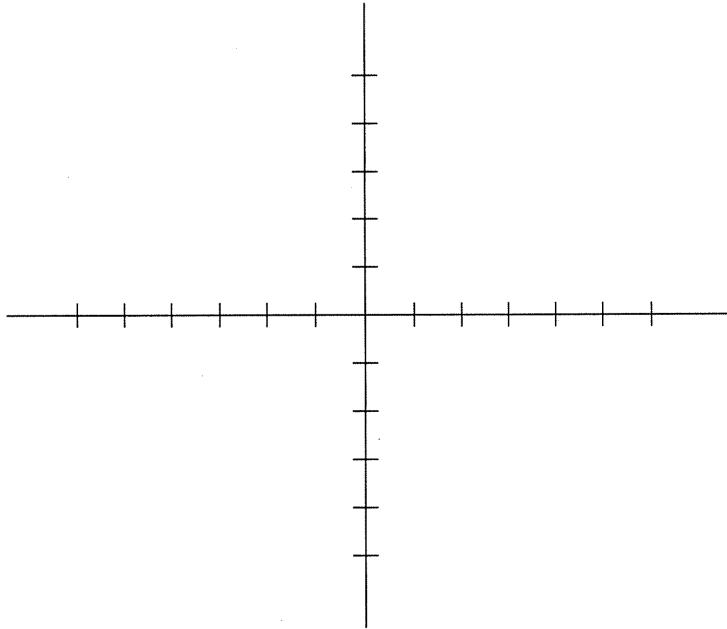
b) Find the exact value of $y(1)$ by solving the initial value problem.

Answer: _____

4. The population y (in thousands) of fish in a lake at a give time t (months) changes at the rate

$$\frac{dy}{dt} = 8y - 2y^2$$

a) (15 points) Sketch, as accurately as you can, a graph of the solution curves for this differential equation. Label the equilibria and indicate whether each is stable or unstable.



b) (5 points) Now suppose that fishing season begins and in addition to the above rate of change, fish are also removed from the lake at the rate of ky thousands of fish per month ($k > 0$). What should k be in order to avoid extinction of the fish population? Be sure to justify your answer.

Answer: _____

5. (20 points) Find the general solution of

$$y''' - 4y' = t^2$$

Answer: _____

6. a) (15 points) Find the general solution of

$$(t^2 + 3)y'' - 2y = 0$$

using a power series about the point $t_0 = 0$. Find the recurrence relation and the first four nonzero terms of each of the two linearly independent solutions.

Answer: _____

b) (5 points) Determine a lower bound for the radius of convergence of the answer to a).

Answer: _____

7. (10 points each) Consider the differential equation

$$y'' - 2y' + 17y = 0$$

a) Find the general solution.

Answer: _____

b) Transform this equation into a system of first order equations and find the general solution of the system.

Answer: _____

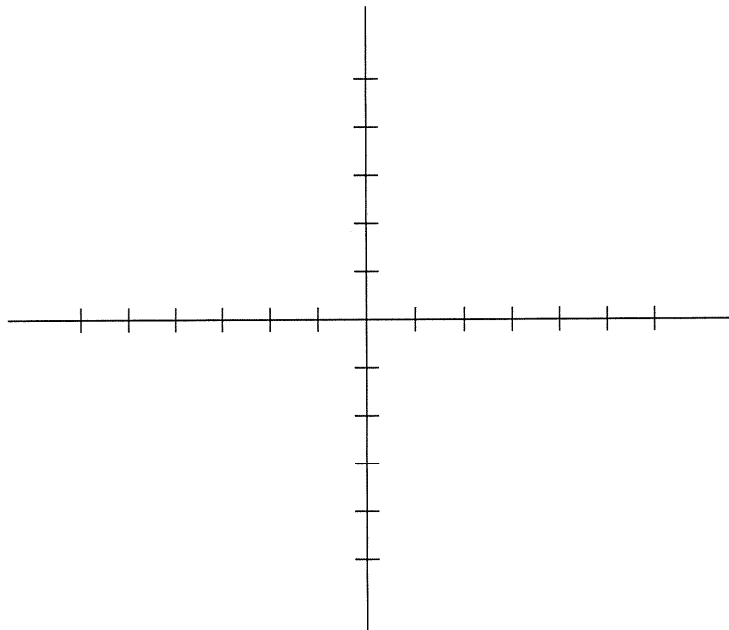
8. (10 points each) Consider the system

$$\begin{aligned}y_1' &= 2y_1 - y_2 \\y_2' &= 4y_1 - 3y_2\end{aligned}$$

a) Find the general solution.

Answer: _____

b) Sketch a few trajectories. Be sure to indicate the direction of motion as $t \rightarrow \infty$.



9. (20 points) Find the general solution of the system

$$\vec{y}' = \begin{pmatrix} 0 & 0 & 1 \\ -3 & 5 & -1 \\ -1 & 0 & 2 \end{pmatrix} \vec{y}$$

Answer: _____

10. (5 points each) For each of the following groups of functions, find (if possible) a differential equation of the form

$$y'' + p(t)y' + q(t)y = 0 \quad p(t), q(t) \text{ continuous}$$

for which the given functions are solutions. If such an equation cannot exist explain why.

a) $y_1 = t, y_2 = t + 1$

Answer: _____

b) $y_1 = \sin \pi t, y_2 = \cos \pi t$

Answer: _____

c) $y_1 = e^t, y_2 = e^{2t}, y_3 = e^{3t}$

Answer: _____

d) $y_1 = t^2$

Answer: _____