Math 215-0 Winter 2003

Final Exam

(200 points - 40% of grade)

Name:

Problem	Max. Points	Score
#1	30	
#2	30	
#3	30	
#4	30	
#5	40	
#6	40	
Total	200	

1. (30pts) Sketch the region of integration (shade it in) AND evaluate by reversing the order of integration.

$$\int_0^1 \int_{\sqrt{y}}^1 \cos x^3 \, \mathrm{d}x\mathrm{d}y$$

2. (30pts) Let T be a triangular lamina with vertices at $(0,0)$, $(2,0)$ and $(1,1)$ and density $\rho(x,y) = (x^2+y^2)^{-1/2}$. Express the mass of T as a double integral in polar coordinates. (You do not need to evaluate.)

3. (30pts) Find the volume of the region E that lies inside the sphere $x^2+y^2+z^2=4$ and between the cones $z=\pm\sqrt{x^2+y^2}$. (Note: the intersection of E and the plane $z=0$ is a disk, NOT a point.)

4. (30pts) Let $\mathbf{F} = (1 + 2xy)e^z\mathbf{i} + x^2e^z\mathbf{j} + (x^2y + x)e^z\mathbf{k}$ and L the oriented line segment from $(1, 0, 0)$ to $(1, 0, 2\pi)$.

a) Compute $\int_L \mathbf{F} \cdot d\mathbf{r}$

b) Let C be the part of the helix $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$ which lies between the planes $z = 0$ and $z = 2\pi$. Explain why $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_L \mathbf{F} \cdot d\mathbf{r}$.

5. (40pts) Let $\mathbf{F} = (x^3 + y\sin z)\mathbf{i} + (y^3 + z\sin x)\mathbf{j} + 3z\mathbf{k}$ and S the surface of the solid bounded by the hemispheres $z = \sqrt{4 - x^2 - y^2}$, $z = \sqrt{1 - x^2 - y^2}$ and the plane $z = 0$. Compute

$$\int\int_S \mathbf{F} \cdot \mathrm{d}\mathbf{S}$$

6. (40pts) A coffee filter occupies the part of the cone $z = \sqrt{x^2 + y^2}$ that lies inside the cylinder $x^2 + y^2 = 1$. Find the center of mass of the filter if the density (in units of g/cm^2) is uniform.