Northwestern University

Name: _____

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Math 214-3, Final Exam

Spring Quarter 2001

Thursday, June 7, 2001

Instructions:
Show *all* your work on these sheets. Feel free to use the opposite side. Make sure that your final answer is clearly indicated. No calculators, books, notes, etc. are allowed. Good luck!

1. (a) Write an equation of the line tangent to the parametric curve $x = \cos^3 t, y = \sin^3 t$ at $t = \frac{\pi}{4}$.

   (b) Find the arc length of the given curve when $0 \leq t \leq \frac{\pi}{4}$.

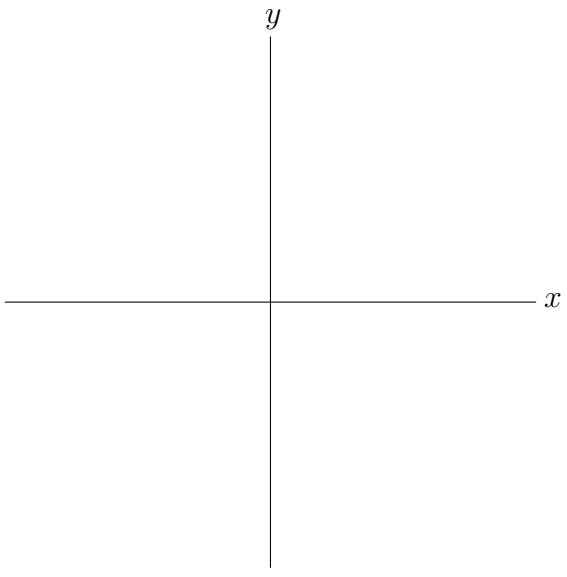
2. Suppose that a dove is flying so that its acceleration vector at time t is given by $\mathbf{a}(t) =< \sin(2t), \cos(2t), 1 >$, its initial velocity is $\mathbf{v}(0) =< 0, 1, 1 >$, and its initial position is $\mathbf{r}(0) =< 1, -1, 0 >$.

   (a) Find the speed of the dove when $t = 1$.

   (b) Find the position vector $\mathbf{r}(t)$ of the dove at time t.

3. Sketch the curve defined in polar coordinates by the equation

   $r = 1 + 2\sin\theta,\ 0 \leq \theta \leq 2\pi$.



4. Let $h(x, y) = x^3 + y^3 + 3xy + \dfrac{1}{8}$

   (a) Determine all local maxima, minima, and saddle points of $h(x, y)$. Are the local extrema also global extrema?

   (b) Match the function $h(x, y)$ in part (a) to the plot of its level curves. Explain.

5. Let S be the surface $x^2 + y^2 - z^2 = 1$.

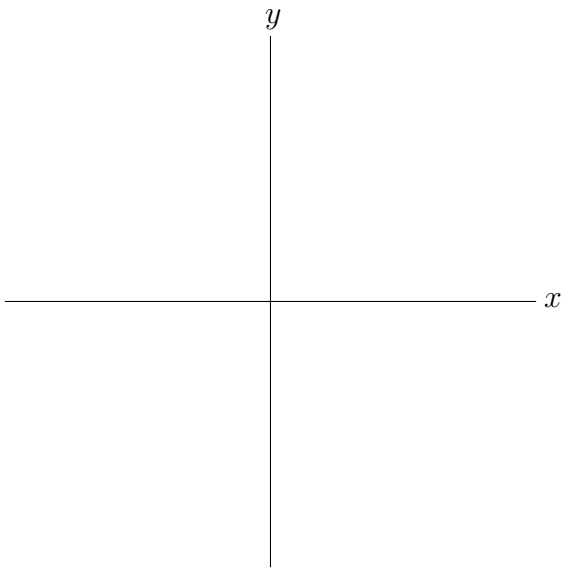
   a. Sketch and label the traces of S in the coordinate planes.

   b. Sketch and label the traces of S in the horizontal planes $z = 0$, 1 and 2.

   c. Sketch and label the surface S itself.

   d. Find the equation of the tangent plane of S at $(1, 1, 1)$.

6. Find the largest domain of definition for the function $f(x, y) = \sqrt{4 - x^2 - y^2}$ and sketch level curves for $z = 0, 1$ and 2 on the same set of axis.



7. Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{\sqrt{x^2 + y^2}}$$

   if it exists, or show that the limit does not exist. Hint: Use polar coordinates.

8. The planes $P_1 : x - 2y + 3z = 0$ and $P_2 : 4x + y + z = 0$ intersect in a line.

   (a) Find parametric equations for that line.

   (b) Find parametric equations for the line through the point $(-3, -3, 1)$ that does not intersect either plane.

9. Use Lagrange multipliers to find the point on the surface $xy + 1 - z = 0$ closest to the origin.

10. Given the three points $P = (1, 0, 2)$, $Q = (2, 1, -3)$, and $R = (3, -2, 0)$, find:

    (a) The equation of the plane containing the points P, Q and R.

    (b) The area of the triangle PQR.

    (c) The height of the triangle PQR when the base is PQ; i.e. find the distance from the point R to the line through P and Q.

11. Suppose that $T(x, y) = x^2 + 2y^2 - x$ is the temperature at the point (x, y). Suppose you are standing at the point $(-2, 1)$.

    (a) You decide to proceed in the direction of the point $(1, -3)$. What will be the rate of change of the temperature? Will the temperature increase or decrease?

    (b) In what direction does the temperature increase most rapidly?

    (c) In what direction does the temperature remain the same?

12. Let $z = x + y$, and let x and y be functions of s and t with $x(0, 0) = 1, y(0, 0) = 2, \frac{\partial x}{\partial s} = 3$, and $\frac{\partial y}{\partial s} = 4$ at $(s, t) = (0, 0)$. Find $\frac{\partial z}{\partial s}$ when $(s, t) = (0, 0)$.

13. Use linear approximation and the fact that $\sqrt[3]{8}\ \sqrt{16} = 8$ to estimate:

$$\sqrt[3]{7}\ \sqrt{17}$$