

Math 214-3, Practice Problems for Test I

Vectors, Cross Product , Lines and planes

1. For which values of the scalar c are the vectors $2c\mathbf{i} - 4\mathbf{j}$ and $3\mathbf{i} + (c - 1)\mathbf{j}$ perpendicular?
2. (a) Find parametric equations of the line L of intersection of the two planes $\mathcal{P}_1 : 3x + 2y + z = 3$ and $\mathcal{P}_2 : -2x + z = 4$.
(b) Find the angle of intersection between the two planes.
(c) Find an equation for a plane which is simultaneously perpendicular to both of the planes of part (a), and which passes through the point $(0, 1, 2)$.
3. Consider the three points $P = (1, 2, -1)$, $Q = (-2, 1, 3)$, and $R = (1, 0, -1)$.
 - (a) Find a parametric equation of the line \mathcal{L}_1 that passes through the points P and Q .
 - (b) Determine whether or not the line \mathcal{L}_1 is parallel, intersects or is skew to the line $\mathcal{L}_2: x = s, y = -s, z = s$.
 - (c) Find the area of the triangle ΔPQR
 - (d) Compute the interior angle of the triangle at P .
 - (e) Compute $proj_{\vec{PQ}}\vec{PR}$.
 - (f) Find the volume of the parallelepiped spanned by the vectors \vec{PQ}, \vec{PR} and the vector \mathbf{i}
 - (g) Write an equation of the plane which passes through the points P, Q and R .
 - (h) Sketch the plane which passes through the points P, Q and R . Hint: Find the x,y and z-intercepts of the plane.
4. Determine the equation of the plane which contains the points $P = (1, 2, -1)$ and $Q = (-2, 1, 3)$ and is perpendicular to the plane $x + 3y - 2z = 3$. Draw a picture and explain!
5. Find an equation for the plane which passes through the point $P = (1, 3, -2)$ and which contains the line of intersection of the planes $x - y + z = 1$ and $x + y - z = 1$.
6. Find the distance between the parallel planes $2x + 3y - z = 1$ and $-4x - 6y + 2z = 2$.

Polar, cylindrical and spherical coordinates

7. Sketch the graph of $r = 2 + 3 \cos \theta$.
8. (a) Consider the “rose” given as the graph of $r = \sin 3\theta$ in polar coordinates. Do the points with $0 \leq \theta \leq \pi$ trace out the whole graph? How many petals does it have?
(b) How about the “rose” given as the graph of $r = \sin 2\theta$ in polar coordinates. Do the points with $0 \leq \theta \leq \pi$ trace out the whole graph? How many petals does it have?
9. Find the polar equation for the graph of the circle of radius 5 centered at the point $(3, 4)$.
10. Convert $(1, \pi/4, \pi/4)$ from spherical to rectangular coordinates.
11. Convert $(1, 1, \sqrt{2})$ from rectangular to spherical coordinates.
12. Find rectangular coordinates for the surface whose equation in cylindrical coordinates is $r = \sin \theta$ and describe this surface.
13. Describe in words or sketch the surfaces represented by
 - (a) $\rho = 2$
 - (b) $\Phi = \pi/4$
 - (c) $\Theta = \pi/2$
14. Describe in words or sketch the solid represented in cylindrical coordinates by the inequalities $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{3}, 0 \leq r \leq 2, -1 \leq z \leq 1$.

Functions and Surfaces

15. Find the domain of the function $f(x, y) = \frac{\sqrt{y - x^2}}{x - 1}$.
16. Identify the traces in the coordinate planes of the quadric surfaces given below. Describe in words or sketch the surfaces.
- (a) $4x^2 + z^2 = y^2$
 - (b) $x^2 + \frac{y^2}{4} - z^2 = 1$
 - (c) $x^2 - \frac{y^2}{4} - z^2 = 1$
 - (d) $x^2 - 2x + \frac{y^2}{4} + z^2 = 0$
 - (e) $z = 2 - r^2$

Curves in space, arclength

17. The parametric equations of a moving point are

$$x(t) = 2 \sin 3t, \quad y(t) = -8t, \quad z(t) = 2 \cos 3t.$$

- (a) Find its velocity, speed, and acceleration at all times.
 - (b) Find the distance travelled by the point from $t = 0$ to $t = 5$.
 - (c) Find the equation of the tangent line to the curve at the point when $t = 0$.
18. A body moves in space so that at time t the acceleration of the body is given by the vector $\mathbf{a}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} - g\mathbf{k}$, where g is a *constant* scalar value. At $t = 0$ the body is at the origin and has velocity $4\mathbf{j}$.
- (a) What is the position of the body when $t = \frac{\pi}{2}$?
 - (b) What is the position of the body when $t = \pi$?
19. Batman drives his rocket powered car over the edge of a cliff. His initial position at the edge of the cliff is $P = (0, 80)$. His initial velocity vector is $\mathbf{v}(0) = 16\mathbf{i}$. His acceleration vector is given by $\mathbf{a}(t) = 20\mathbf{i} - 10\mathbf{j}$.
- (a) Find Batman's position vector $\mathbf{r}(t)$.
 - (b) How long before Batman crashes into the x -axis (i.e., the ground)?
 - (c) What is Batman's speed when he hits the ground?
20. A ball is thrown at an angle of 45° to the ground. If the ball lands 90 m away, what was the initial speed of the ball?
21. Suppose C is the curve given by the vector function $\mathbf{r}(t) = \langle t, t^2, 1 - t^2 \rangle$. Find the unit tangent vector, the unit normal vector at the point where $t = 1$.