Math 214-3, Practice Problems for Test II

Projectile Motion, Curvature

1. A canon is fired with an angle of elevation of 45 degrees. The speed of the canonball as it leaves the muzzle is 128 ft/sec. On Earth the acceleration due to gravity is 32 ft/sec^2 downward.
   (a) What is the maximum height of the cannonball?
   (b) How long is the canonball be in the air?

2. Find the curvature κ of the curve $\vec{\mathbf{r}}(t) = \langle 2\sin t, 3t, 2\cos t\rangle$ when $t = \frac{\pi}{2}$.

Parametric Surfaces

3. Describe in words the parametric surfaces with vector equations:
   (a) $\vec{\mathbf{r}}(u, v) = (2u + 3v)\mathbf{i} + (u)\mathbf{j} + (v)\mathbf{k}$
   (b) $\vec{\mathbf{r}}(u, v) = \cos u \sin v\mathbf{i} + \sin u \sin v\mathbf{j} + \cos v\mathbf{k}, 0 \le u \le 2\pi, 0 \le v \le 2\pi$. Hint: Find $x^2 + y^2$.

4. Find a parametric representation of the surface consisting of the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 4$.

Functions, Partial Derivatives

5. Let $f(x, y) = \sqrt{x^2 + y^2 - 1}$.
   (a) Find the domain of the function f.
   (b) Sketch the level curves $f(x, y) = k$ for $k = 0, 1, 2$, and 3.
   (c) Sketch the surface $S : z = f(x, y)$.
   (d) Find the equation of the tangent plane to S at $(1, 2, 2)$.

6. Explain why the following limit does not exist or find the limit, if it exists:

$$\lim_{(x,y)\to(0,0)} \frac{5x^2 - y^2}{x^2 + y^2}$$

7. Find the value of $\frac{\partial z}{\partial x}$ at $(1, 1, 1)$ if the equation

$$xy + z^3x - 2yz = 0$$

   defines z implicitly as a function of the two independent variables x and y and the partial derivative exists.

8. Show that if $w = f(u, v)$ satisfies the Laplace equation $f_{uu} + f_{vv} = 0$, then with

$$u = \frac{x^2 - y^2}{2} \quad \text{and} \quad v = xy,$$

   w satisfies the Laplace equation $w_{xx} + w_{yy} = 0$.

9. Consider the function $f(x, y) = \dfrac{1}{1 + x + y}$. Use differentials to find the approximate value of $f(3.02, 6.05)$.

10. Sand is pouring onto a conical pile in such a way that at a certain instant the height is 100 inches and increasing at 3 inches per minute and the radius is 40 inches and increasing at 2 inches per minute. How fast is the volume increasing at that instant?

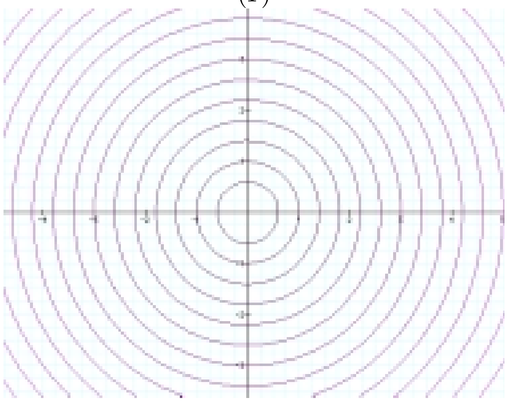
11. Describe the contour curves of the function $f(x, y) = \sqrt{1 - x^2 - 2y^2}$.

12. Match the functions to its contour curves.

(A) $f(x,y) = 3 - x^2 - y^2$ (B) $f(x,y) = \sin x$

(C) $f(x,y) = \sqrt{x^2 + y^2}$ (D) $f(x,y) = x + y$

(I)



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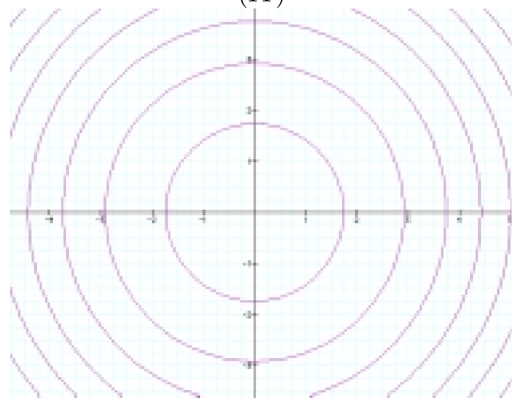

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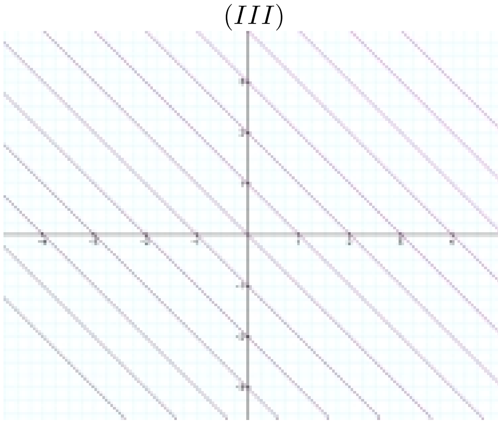

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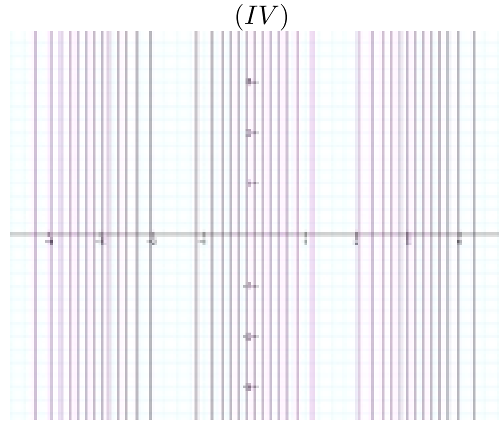
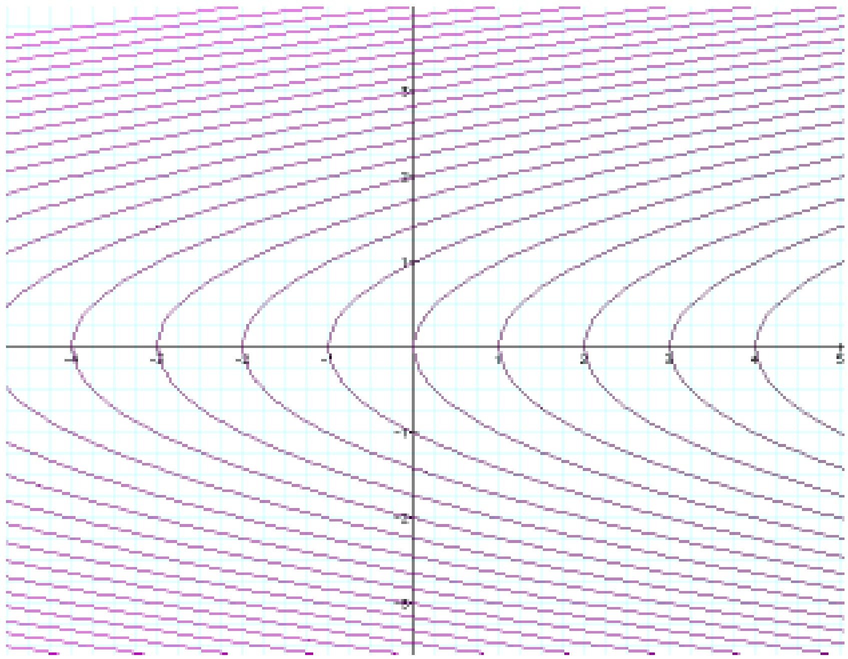


13. Shown in the figure below is a contour map of a function $f(x, y)$. Suppose $f(0, 0) = 0$, $f(1, 0) = 1$, and $f(x, 0) = x$. Estimate $f_x(1, 1)$ and $f_y(1, 1)$ as best you can. Decide whether or not $f_{xx}(1, 1)$, $f_{xy}(1, 1)$, and $f_{yy}(1, 1)$ are positive, negative, or zero.



Line Integrals, for engineering sections only

14. Find the work done by the force $\mathbf{F} = y\mathbf{i} - z\mathbf{j} + x\mathbf{k}$ in moving a particle along the straight line segment from the point $(3, 1, 0)$ to the point $(2, 0, 1)$. The units are in lbs and feet.

15. Let $\mathbf{F}$ be the force field shown below. Determine whether the work counterclockwise along the circle of radius 3 is positive, negative, or zero.