Northwestern University

Name: ______________________

Id #: ______________________

Math 214-3 Final Exam

Spring Quarter 2003

Monday, June 9, 2003

Check your instructor's name and section:

Liang 10:00		Bode 11:00	
Myung 12:00		Liang 1:00	
Myung 1:00			

Prob.	Possible points	Score
Part I		
1	21	
2	4	
3	3	
4	8	
5	8	
6	6	
Part II		
1	6	
2	12	
3	6	
4	12	
5	16	
6	8	
7	10	
8	8	
9	16	
10	16	
11	12	
12	10	
13	8	
14	10	

Instructions:
Show *all* your work on these sheets. Feel free to use the opposite side. Make sure that your final answer is clearly indicated. No calculators, books, notes, etc. are allowed. Good luck!

Prob.	Possible points	Score
Part I	50	
Part II	150	
TOTAL	200	

Part I, Multiple Choice

Circle the correct answers. There will be no partial credit for problems in part I.

1. (21 points) Determine whether the statement is true or false.

   (a) Among the expressions $\vec{a}\cdot(\vec{b}\cdot\vec{c})$, $\vec{a}\cdot(\vec{b}\times\vec{c})$, $\vec{a}\times(\vec{b}\cdot\vec{c})$, and $(\vec{a}\cdot\vec{b})\times(\vec{c}\cdot\vec{d})$, only $\vec{a}\cdot(\vec{b}\times\vec{c})$ is a well-defined scalar.

   True False

   (b) The two planes $p_1 : 3x+y-z=2$ and $p_2 : x-y+2z=-5$ are perpendicular to each other.

   True False

   (c) The equation $\vec{r}(u,v)=u\cos(v)\vec{i}+u\sin(v)\vec{j}+v\vec{k}$ represents a cone.

   True False

   (d) If $(0,0)$ is a point on the level curve $f(x,y)=0$, then $\nabla\mathbf{f}(0,0)$ is a vector perpendicular to the tangent line at (0,0) to the curve.

   True False

   (e) Two level curves of the function $f(x,y)$ corresponding to different values of f cannot ever cross.

   True False

   (f) If $\nabla\mathbf{f}=\nabla\mathbf{g}$, then $f=g$.

   True False

   (g) For any vectors $\vec{a}$ and $\vec{b}$ in 3-dimensional space, $(\vec{a}\times\vec{b})\cdot\vec{c}=0$ implies that $\vec{a}$ and $\vec{c}$ are parallel.

   True False

2. (4 points) Of the following, which is NOT an equation of a plane?

   (a) $\theta = \pi$ (cylindrical coordinates)

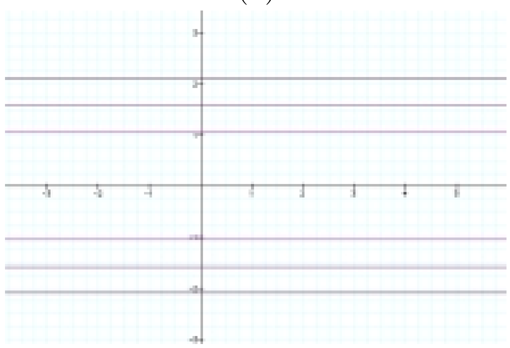
   (b) $z = -x + 3y$

   (c) $z = r$ (cylindrical coordinates)

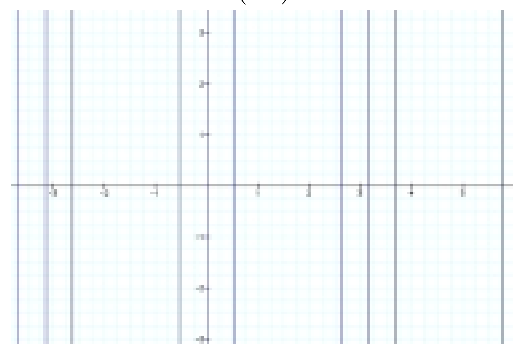
   (d) $\vec{r}(u, v) = (3u + v - 1)\vec{i} + (u - v)\vec{j} + v\vec{k}$

3. (3 points) Which of the following is a contour diagram for $f(x, y) = \sin x$?

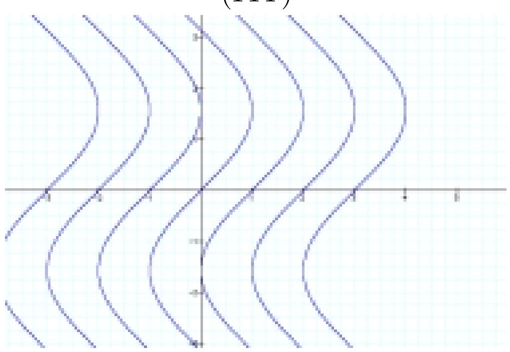
(I)



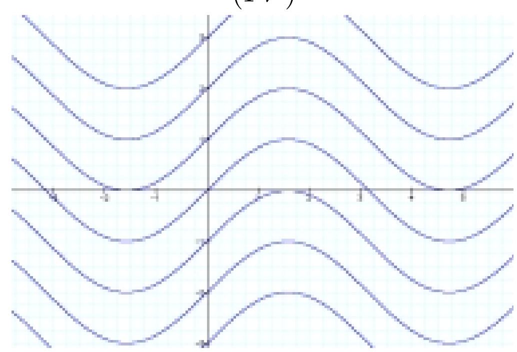
(II)



(III)



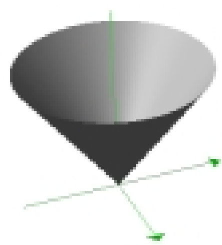
(IV)

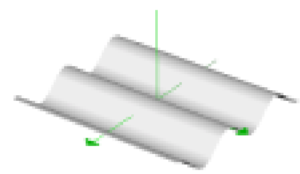


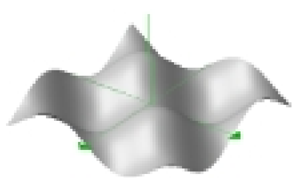
4. (8 points) Match the functions to its surfaces.

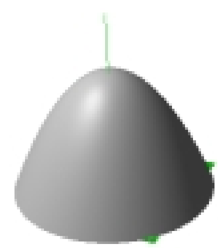
(A) $z = 1 - r^2$ (B) $z = \sin x \sin y$

(C) $z = \sqrt{x^2 + y^2}$ (D) $z = \sin x \cos x$









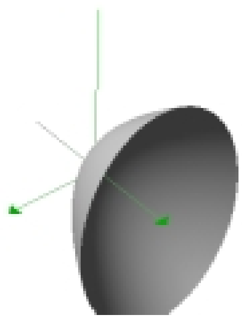
5. (8 points) Match the parametric surfaces to its vector equations.

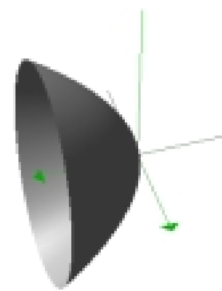
(a) $\vec{\mathbf{r}}(u,v) = (u\sin v)\mathbf{i} + (u\cos v)\mathbf{j} + (u)\mathbf{k}, 0 \le v \le 2\pi, 0 \le u \le 2$

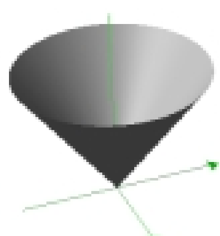
(b) $\vec{\mathbf{r}}(u,v) = (u^2)\mathbf{i} + (u\cos v)\mathbf{j} + (u\sin v)\mathbf{k}, 0 \le v \le 2\pi, 0 \le u \le 2$

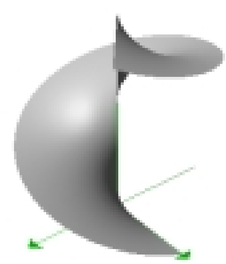
(c) $\vec{\mathbf{r}}(u,v) = (u\cos v)\mathbf{i} + (u^2)\mathbf{j} + (u\sin v)\mathbf{k}, 0 \le v \le 2\pi, 0 \le u \le 2$

(d) $\vec{\mathbf{r}}(u,v) = (u\sin v)\mathbf{i} + (u\cos v)\mathbf{j} + (v)\mathbf{k}, 0 \le v \le 2\pi, 0 \le u \le 4$









6. (6 points) You are in an 80^o F classroom. Temperatures outside are in the upper 60's. Deciding that it's too hot in the classroom, you convince your instructor to open a window. Let $z = T(t, x)$ be the temperature in the room, t minutes after the window was opened, x feet from the window.

   Determine whether the statement is true or false.

   (a) $T_t(1, 1)$ is positive.

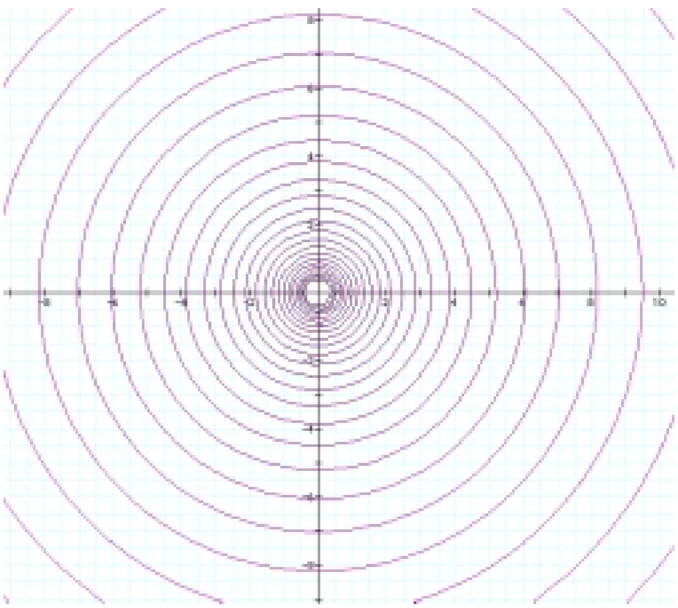
   True False

   (b) $T_x(1, 1)$ is positive.

   True False

Part II

1. (6 points) A contour map of a function f is shown. Use it to make a rough sketch of the graph f.



2. (12 points) A child is sliding down a helical slide. Her position at time t after the start is given by $\vec{\mathbf{r}}(t) = \cos t\ \mathbf{i} + \sin t\ \mathbf{j} + (10 - t)\ \mathbf{k}$

   (a) At time $t = 2\pi$, the child leaves the slide on the tangent to the slide at that point. What is the equation of the tangent?

   (b) What was the child's speed when leaving the slide?

   (c) What far did the child slide before leaving the slide at time $t = 2\pi$?

3. (6 points) Sketch the curve defined in polar coordinates by the equation
$r = 1 + \sin\theta,\ 0 \leq \theta \leq \pi.$

4. (12 points) Let $f(x, y) = x^2 + y^2 + 2x^2y + 3$.

   Determine all local maxima, minima, and saddle points of $f(x, y)$.

5. (16 points) Let $f(x, y) = 2\sqrt{1 - x^2 - y^2}$.

(a) Find the domain and range of the function $f(x, y)$.

(b) Sketch the level curves for the function $f(x, y)$ for the values $k = 0, 1, 2$.

(c) Sketch the traces of the surface $S : z = f(x, y)$ in the $yz-$ and $xz-$ planes.

(d) Sketch the surface S.

6. (8 points) Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{x^2+xy}{\sqrt{x^2+y^2}}$$

if it exists, or show that the limit does not exist. Hint: Use polar coordinates.

7. (10 points) Consider the surface $xyz + x^2 = y^2 + 2$.

   Find the equation of the tangent plane at $(3, 1, -2)$.

8. (8 points) The planes $P_1 : x - 2y + 3z = 0$ and $P_2 : 4x + y + z = 0$ intersect in a line.

   Find parametric equations for that line.

9. (16 points) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y, z) = x - y + z$ on the sphere $x^2 + y^2 + z^2 = 3$.

10. (16 points) Given the four points $P = (0, 0, 1)$, $Q = (2, 0, 0)$, $R = (0, 3, 0)$ and $S = (1, 1, 1)$, find:

(a) The equation of the plane $\mathcal{P}$ containing the points P, Q and R.

(b) The area of the triangle PQR.

(c) The volume of the pyramid with vertices P, Q, R and S. The volume of a pyramid is $1/6$ of the volume of the corresponding parallelepiped.

(d) The distance from the point S to the plane $\mathcal{P}$

11. (12 points) Suppose that the elevation on a hill is given by $f(x, y) = 200 - y^2 - 4x^2$, units are in feet.

(a) A climber is standing at the point at $(1, 2, 194)$. In which direction should the climber proceed initially in order to reach the top of the hill fastest?

(b) If a level road is to be built at elevation 100 feet, find the shape of the road.

(c) Suppose a climber decides to proceed from the point $(1, 2, 194)$ in the direction of the point $(4, 1, 135)$. At what rate is the elevation changing? Hint: Find the directional derivative at the point $(x, y) = (1, 2)$ in the direction of the point $(4, 1)$.

12. (10 points) Let $z = 3x\cos y$, and let $x = s^2 + t^2$ and $y = t/s$, find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ when $(s, t) = (1, 0)$.

13. (8 points) Supppose that a cylindrical can is designed to have radius 1 in. and a height of 5 in., but that the radius and height are off by the amounts $dr = 0.03$ and $dh = -0.1$. Use linear approximations or differentials to estimate the change of volume of the can.

14. (10 points) An athlete throws a shot at an angle of 45^o to the horizontal at an initial speed of 60 ft/s. It leaves his hand 6ft above the ground.

    (a) Where is the shot 2 seconds later?

    (b) How high does the shot go?