

Final

1. (30pts) Consider a triangle with vertices $P(2, 1, 1)$, $Q(1, 0, 3)$, and $R(1, 3, 7)$.
 - (a) Determine whether the angle in the triangle at the point Q is acute ($< 90^\circ$), a right angle, or obtuse ($> 90^\circ$).
 - (b) Compute the area of the triangle PQR .
 - (c) Find the parametric equation of a line through P which is perpendicular to the plane containing the triangle PQR .

2. (10pts) Find the distance from the point $(3, 1, 1)$ to the plane $2x + y - 4z = -1$.

3. (30pts) If

$$\vec{v}(t) = t^3 \vec{i} + e^{2t} \vec{j} + \sin t \cos t \vec{k}$$

represents the velocity of a particle at time t , find the following

- (a) the position vector $\vec{r}(t)$ at time t assuming $\vec{r}(0) = \langle 1, 0, -1 \rangle$,
- (b) the acceleration vector $\vec{a}(t)$,
- (c) the speed of the particle at time $t = \pi$,

4. (20pts) Find the length of the parametric curve $x(t) = t, y(t) = \frac{2}{3}t^{3/2}, 0 \leq t \leq 1$.

5. (20pts) On the planet Krypton the baby Superman jumps with an angle of elevation of 45 degrees. His speed when he leaves the ground is 128 ft/sec . After exactly $\sqrt{2}$ seconds he lands on the ground again.

(a) How far did he jump?

(b) What is the acceleration due to gravity on Krypton?

6. (20pts) Sand is pouring onto a conical pile in such a way that at a certain instant the height is 100 inches and increasing at 3 inches per minute and the radius is 40 inches and increasing at 2 inches per minute. How fast is the volume increasing at that instant? Hint: the volume of a cone is

$$V = \frac{1}{3}\pi r^2 h$$

7. (20pts) If $f(u, v, w)$ is differentiable and $u = x - y$, $v = y - z$, and $w = z - x$, then show that

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0$$

8. (10pts) Shown in the figure is a contour map of a function $f(x, y)$. Estimate $f_x(9, 27)$ and $f_y(9, 27)$ as best you can and sketch the vector $\nabla f(9, 27)$ into the diagram.

9. (20pts) Determine and classify all the critical points of the function
 $f(x, y) = 3x^3 + y^2 - 9x + 4y$.

10. (20pts) Find the maximal values of $f(x, y, z) = x + 2y + 3z$ subject to the constraints

$$x - y + z = 1$$

and $x^2 + y^2 = 1.$