Math 214-2, Review problems for Test # II

1. Determine whether or not the following improper integrals are convergent or divergent. Evaluate those that are convergent.

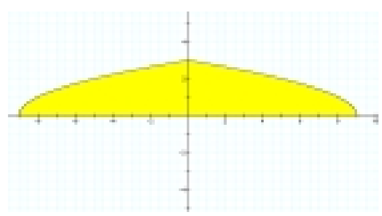
$(a)\ \displaystyle\int_{-\infty}^{\infty} \frac{dx}{x^2+1} \qquad (b)\ \int_0^1 \ln x\ dx \qquad (c)\ \int_0^1 \frac{dx}{x-1}$

$(d)\ \displaystyle\int_e^{\infty} \frac{dx}{x(\ln x)^2} \qquad (e)\ \int_0^{\infty} \frac{e^{\sqrt{x}}dx}{\sqrt{x}} \qquad (f)\ \int_1^{\infty} \frac{dx}{x+e^{2x}}$

2. Let R be the region bounded by the curves $y = 7x, y = 14x$ and $y = 7$. Find the volume of the solid obtained by rotating the region R about the x-axis

3. Let R be the region bounded by the curve $4y = x^2$, and $x = 2y - 4$.
(a) Find the area of the region R, and
(b) find the volume of the solid obtained by rotating the region R about
(i) the x-axis, (ii) the line $x = 5$, and (iii) the line $y = -1$.

4. Use the method of Shells to set up the integral that gives the volume of the solid generated by rotating the region above the x-axis bounded by $x = 9 - y^2$ and $x = y^2 - 9$ around the line $y = -2$.



5. Consider the pairs of parametric equations $x = 2 - 3t, y = 7 - 6t$ and $x = t - 1, y = 1 + 2t$. Show that these pairs of equations produce the same line.

6. Find the arc length of the curve $y = 2x^{3/2}$ between $x = 0$ and $x = 3$.

7. A curve is written parametrically by $x = 4\cos 3t$ and $y = 4\sin 3t$. Describe this curve and compute the length of the curve for $0 \le t \le \frac{2}{3}\pi$.

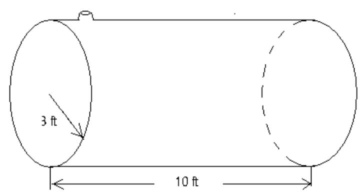
8. The temperature (in^{o}C) of a metal rod 5m long is $4x$ at a distance x meters from one end of the rod. What is the average temperature of the rod?

9. A rope weighing 0.5 kg/m is used to draw a 10 kg bucket of water from a well 30 m deep. How much work is required to pull the bucket to the top of the well?

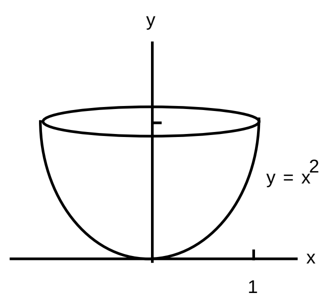
10. The Great Cone of Haverford College is a monument built by freshmen during a customs week long, long ago. It is 100 ft. high and its base has a diameter of 100 ft. It has been built from bricks (purportely made of straw) which weigh 2

lbs/ft ^{3}. Use a definite integral to approximate the amount of work reqired to build the Cone.

11. A gasoline tank shown below is half full of gasoline. Find the work required to pump the gas out of the outlet. Assume the gasoline weighs 45 lb per cubic foot.

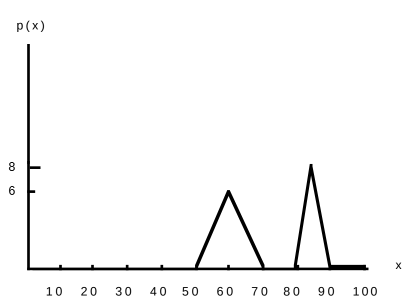


12. An object has the shape drawn below. Its boundary is the result of rotating the curve $y = x^2$ (for $0 \le x \le 1$) around the y-axis. (Units are in centimeters.)



(a) Find the total volume of this object.
(b) Suppose that the density of this object varies with height according to the rule $\rho(y) = 8(2 - y)$ (grams per cubic centimeters) at height y, find the total weight of the object.

13. The scores on an economics test have the following probability density function (values in percentage): Use the graph below to answer the following questions.



(a) Are there more students with scores between 50 and 70 or between 80 and 90?

(b) Find the following probablities: (i) $P(X < 50)$, (ii) $P(X > 70)$ and (iii) $P(50 < X < 70)$
(c) Calculate the median and mean for this grade distribution.

14. Suppose the waiting time (in minutes) for a customer's call to be answered by a company's representative is modeled by an exponential density function:
$f(t) = \begin{cases} 0 & t < 0 \\ ce^{-ct} & t \geq 0 \end{cases}$
(a) Explain why the function defined above is a probability density function.
(b) Set up, but do not evaluate, an integral that represents the probability a customer has to wait for more than 4 minutes.
(c) Find the number c such that the probability in part (b) is 0.2.

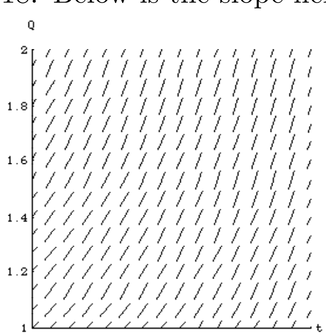
15. Solve the initial value problem.

$$\frac{dy}{dx} = x^2y^2 - y^2, \quad y(0) = 1$$

16. A bacteria population grows at a rate proportional to its size. The count initially was 400 and 25,000 after 4 hours. In how many minutes does the population double?

17. Newton's Law of Cooling states that the rate of change of temperature of an object is proportional to the difference between the temperature of the object and the temperature of the surrounding air.
A detective discovers a corpse in an abondoned building, and finds its temperature to be 27^oC. An hour later its temperature is 21^oC. Assume the air temperature is 8^oC, that normal body temperature is 37^oC, and that Newton's Law of Cooling applies to the corpse.
(a) Write a differential equation satisfied by the temperature H of the corpse at time t. Measure time from the moment the corpse is discovered.
(b) Find a formula for the temperature H of the corpse at time t by solving the differential equation in part (a).
(c) How long has the corpse been dead at the moment of discovery?

18. Below is the slope field for the differential equation $\frac{dQ}{dt} = Q - t$

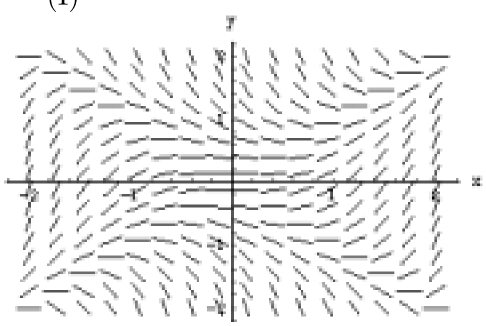


On the graph above of the slope field, sketch the graph of the solution curve through the point $Q = 1.5$, $t = 1$.

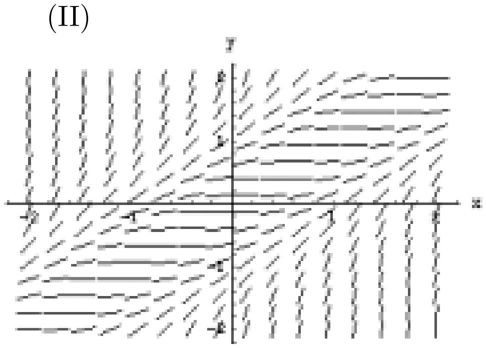
19. The slope fields for several differential equation are graphed below. Determine which equations listed below go with each of the graphs without solving the equations. (Each differential equation matches one graph).

(a) $\dfrac{dy}{dx} = (x-y)^2$ (b) $\dfrac{dy}{dx} = (x+y)^2$ (c) $\dfrac{dy}{dx} = x^2 - y^2$

(I)



(II)



(III)