Northwestern University

Name: ______________________

Id #: ______________________

Math 214-2 Test # 2

Winter Quarter 2003

Wednesday, February 26, 2003

Check your instructor's name and section:

Myung	8:00		Liang	12:00	
Myung	9:00		Song	12:00	
Liang	10:00		Song	1:00	
Bode	11:00				

Prob.	Possible points	Score
Part I		
1-2	8	
3	8	
4	4	
5	4	
Part II		
1	8	
2	10	
3	6	
4	12	
5	10	
6	10	
7	10	
8	10	

Instructions:
Show *all* your work on these sheets. Feel free to use the opposite side. Make sure that your final answer is clearly indicated. No calculators, books, notes, etc. are allowed. Good luck!

Prob.	Possible points	Score
Part I	24	
Part II	76	
TOTAL	100	

Part I, Multiple Choice

Circle the correct answers. There will be no partial credit for problems in part I.

1. (4 points) Find the average value of the function $f(x) = \sin x$ on the interval $[0, \pi]$.

   A) 2π

   B) 2

   C) π

   D) $\pi/2$

   E) $2/\pi$

2. (4 points) Give an integral representing the length of the curve with parametric equations $x = t^3, y = t^4, 0 \leq t \leq 1$.

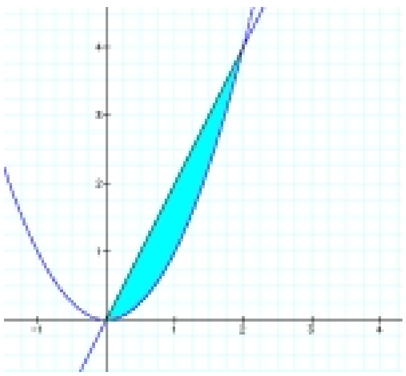
   A) $\int_0^1 \sqrt{(t^3 + t^4)}\, dt$

   B) $\int_0^1 \sqrt{(1 + 3t^2)}\, dt$

   C) $\int_0^1 \sqrt{(9t^4 + 16t^6)}\, dt$

   D) $\int_0^1 \sqrt{(3t^2 + 4t^3)}\, dt$

3. (8 points) Let R be the region bounded by the curves $y = 2x$ and $y = x^2$.



Which of the following integrals represents the volume of the solid obtained by rotating the region R about

(a) the x-axis

A) $\int_0^2 2\pi((2x)^2 - (x^2)^2)\, dx$

B) $\int_0^2 2\pi((2x) - x^2)\, dx$

C) $\int_0^2 \pi((2x)^2 - (x^2)^2)\, dx$

D) $\int_0^2 \pi((x^2)^2 - (2x)^2)\, dx$

(b) the y-axis

A) $\int_0^2 2\pi x((2x)^2 - (x^2)^2)\, dx$

B) $\int_0^2 2\pi x((2x) - x^2)\, dx$

C) $\int_0^2 \pi((x^2) - (2x))\, dx$

D) $\int_0^4 \pi(y - y^2/4)\, dy$

E) $\int_0^4 2\pi(\sqrt{y} - y/2)\, dy$

F) $\int_0^4 2\pi(y - y^2/4)\, dy$

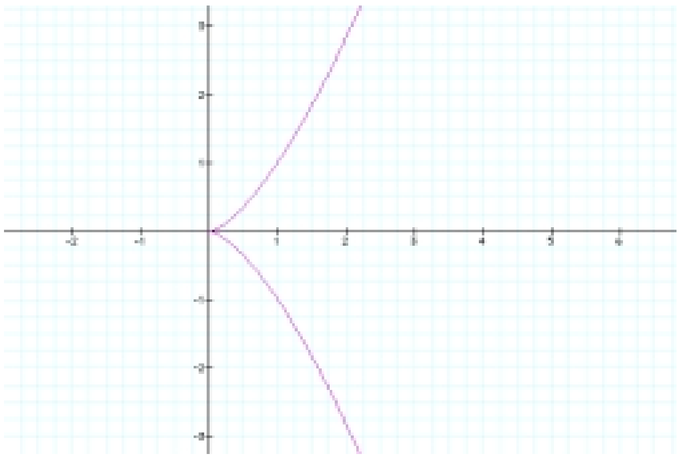
4. (4 points) Match the parametric equations with its graph:

(I) $x = 3\sin t, y = -\cos t$

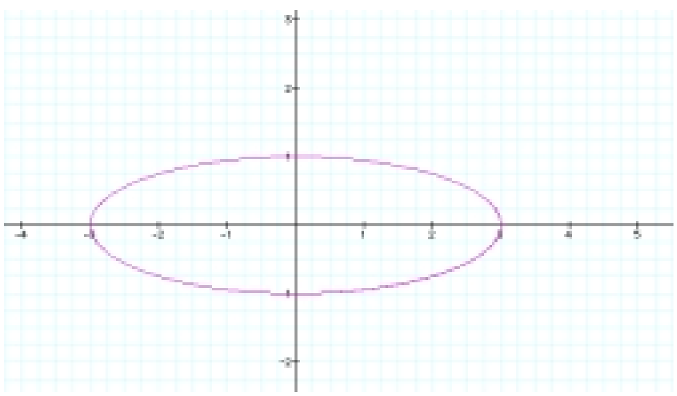
(II) $x = t^2, y = t^3$

(III) $x = \sin t, y = 3\cos t$

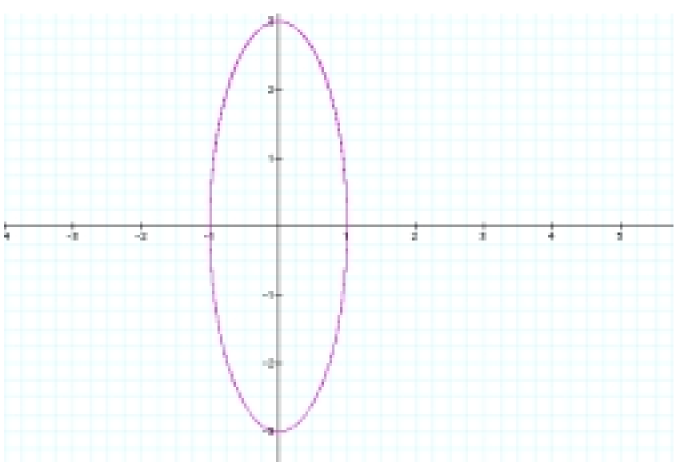
(A)



(B)

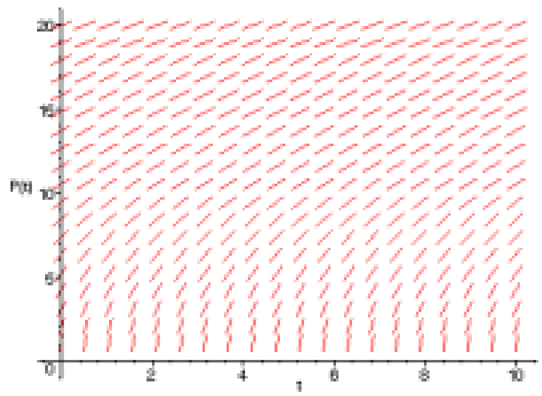


(C)

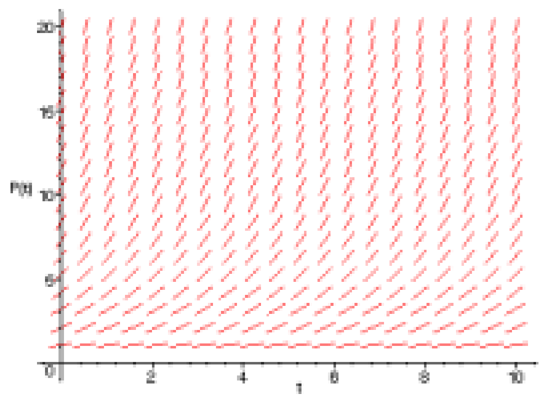


5. (4 points) Which direction field below is the direction field of the differential equation $\dfrac{dy}{dx} = \dfrac{y}{2}$?

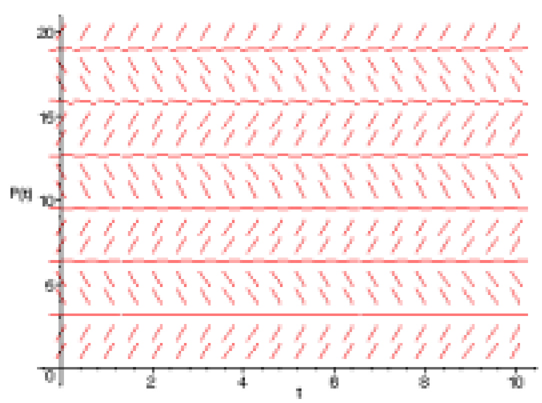
(A)



(B)



(C)

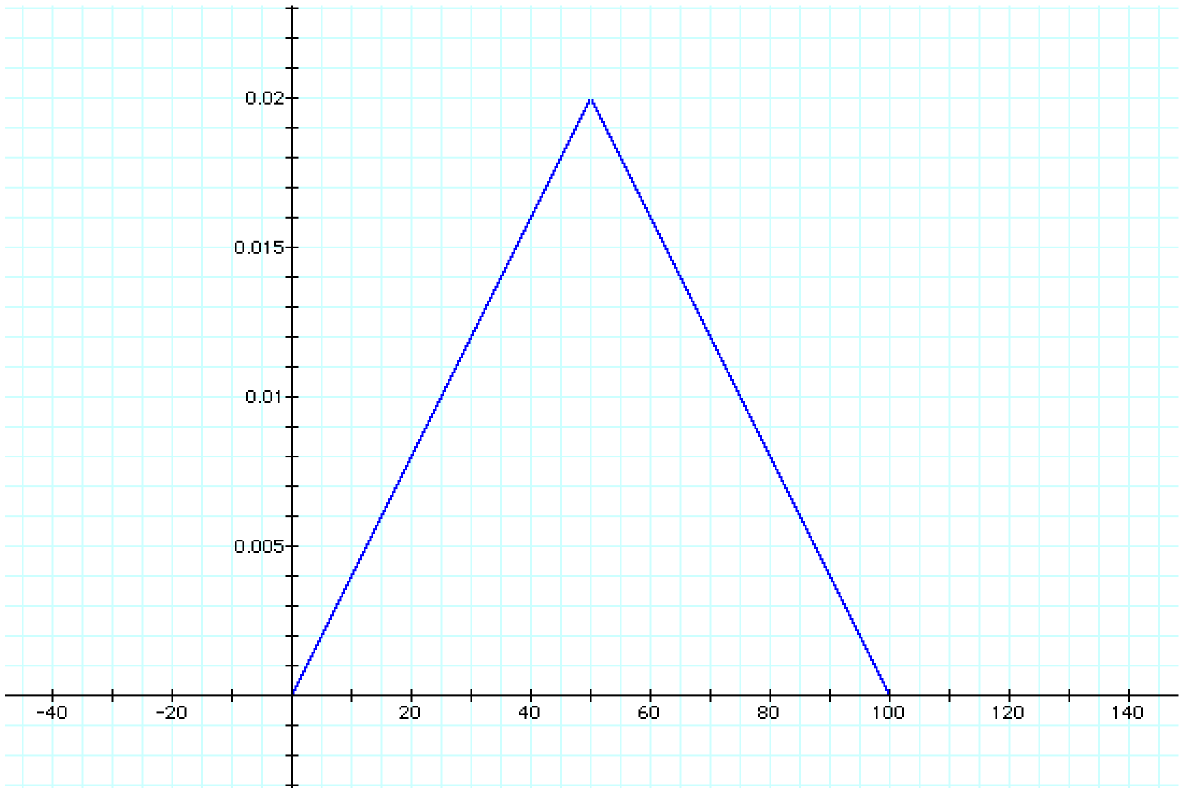


Part II

1. (8 points) Use the Comparison Theorem to determine whether $\int_1^\infty \frac{dx}{x^5+4}$ is convergent or divergent.

2. (10 points) A rescue cable attached to a helicopter weighs 2 lb/ft. A 200 lb man grabs the end of the rope and is pulled from the ocean into the helicopter. How much work is done in lifting the man if the helicopter is 30 ft above the water?

3. (6 points) A professor gives the same 100-point midterm year after year and discovers that the students' scores tend to follow the probability function $p(x)$ pictured below.



(a) What is the probability that a student score will be above 60 points?

(b) Find the median score.

4. (12 points) The function f is defined for all real numbers.

$$f(t) = \begin{cases} e^{-t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

(a) Explain why the function f is a probability density function.

(b) What is the mean for this distribution?

5. (10 points) Calctown has a fixed population of $10,000$ people. Assume that the rate of increase of the number $N(t)$ of people who have had the flu is proportional to the product of the number of people who have had the flu and the number of people who have not had it. t is measured in months after January 1, 2003. On January 1, 2003, the number of people who have had the flu is 1000 and is increasing at a rate of 600 people per month. Determine a differential equation that $N(t)$ must satisfy. Use the given information to find the proportionality constant; i.e. find the value of the constant in your differential equation. You do not need to solve this differential equation .

6. (10 points) Solve the initial value problem.

$$\frac{dy}{dx} = y - 2, \quad y(0) = 10$$

7. (10 points) A population of bacteria is growing according to the differential equation

$$\frac{dP}{dt} = \frac{1}{2}P(1 - \frac{1}{50}P)$$

where P is the population measured in thousands. Below is the direction field for this differential equation

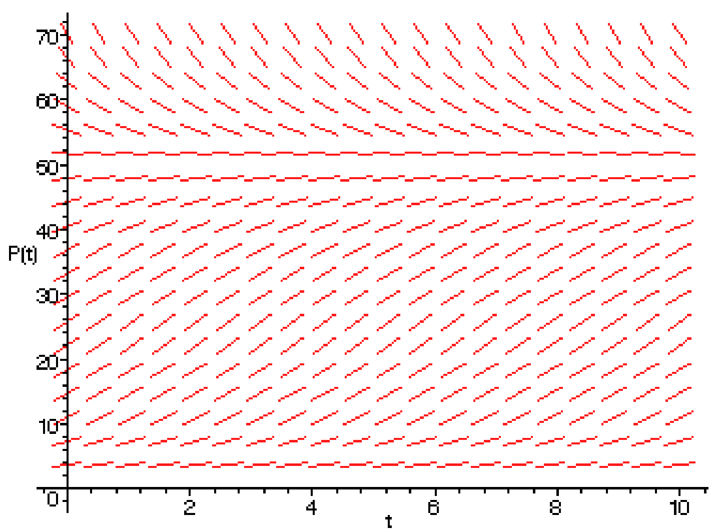
(a) On the graph below, sketch the solution curves

(i) through the point $(0, 30)$ and

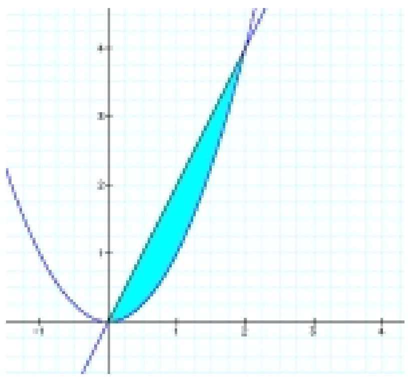
(ii) through the point $(0, 50)$ and

(iii) through the point $(0, 70)$.

(b) In the long run how large will the population of bacteria be?



8. (10 points) Let R be the region bounded by the curves $y = 2x$ and $y = x^2$.



Use the shell method to set up the integral for the volume of the solid obtained by rotating the region R about the line $x = 3$. You do not need to evaluate the integral.