



Math 214-2 Common Final

Fall Quarter 2002

Thursday, December 10, 2002

Check your instructor's name and time:

Wen 9:00 (sec.31)		Lerma 10:00 (sec.41)	
Wen 12:00 (sec.61)		Vilonen 1:00 (sec.71)	

Prob.	Possible points	Score
1	6	
2	8	
3	8	
4	10	
5	10	
6	10	
7	10	
8	20	
9	12	
10	16	
11	10	
12	16	
13	18	
14	16	
15	30	
TOTAL	200	

Instructions:

Show *all* your work on these sheets. Feel free to use the opposite side. Make sure that your final answer is clearly indicated. No calculators, books, notes, etc. are allowed. Good luck!

1. (6 points) Evaluate $\int e^{\sin x} \cos x \, dx$.

2. (8 points) Evaluate $\int x \tan^{-1} x \, dx$.

3. (8 points) Evaluate $\int \frac{x - 1}{x^2 + 3x + 2} dx$.

4. (10 points) Evaluate $\int \sin(\ln t) dt$.

5. (10 points) Evaluate the improper integral $\int_2^{\infty} \frac{1}{x \ln x} dx$ or show that it is divergent.

6. (10 points) Use Simpson's Rule with 4 intervals to find the integral $\int_0^1 f(x) dx$, where the values of f are given by the table below.

x	0.00	0.25	0.50	0.75	1.00
$f(x)$	2.00	1.00	2.00	3.00	5.00

7. (10 points) Find the area of the region bounded by $y = 5x - x^2$ and $y = x$.

8. Consider the region R bounded above by $y = \frac{\pi}{2}$ and below by the curve $\sin y = \sqrt{x}$, as shown below.

Suppose we rotate this region about the x -axis to make a solid, and want to find the volume.

- a) (10 points) Set up the integral to find the volume using washers.

b) (10 points) Set up the integral to find the volume using shells.

9. (12 points) Solve the following initial value problem:

$$\begin{cases} \frac{dy}{dx} = 3x^2(y^2 + 1) \\ y(0) = 1 \end{cases}$$

You must solve for y .

10. Consider the series $\sum_{n=3}^{\infty} \frac{(-1)^n}{\ln(\ln n)}$.

a) (8 points) Does the series converge or diverge? Justify your answer.

b) (8 points) Does the series converge absolutely? Justify your answer.

11. (10 points) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x+1)^n}{n2^n}$.

- 12.** (16 points) Using the power series representation for $\frac{1}{x+1}$, find the power series representation of $x \ln(x+1)$ which is valid for $|x| < 1$.

13. (18 points) Evaluate $\int \ln(1 + x^5) dx$ as a power series.

14. (16 points) Find the Taylor series for $f(x) = \cos x$ at $\pi/3$.

- 15.** When ice forms on the lake, the rate of the increase of the thickness of the ice is inversely proportional to the thickness of the ice.
- a) (8 points) Denoting the thickness of the ice by T and the proportionality constant by k , write down a differential equation which models this situation.

b) (10 points) Solve the differential equation in (a).

c) (12 points) If the thickness of the ice is 1 mm at midnight and 2 mm at 6 AM when is the thickness of the ice 3 mm?