Math 214-1, Midterm Review Problems

1. Let $f(x) = \ln x, g(x) = \sqrt{x-1}$. Find the domains of definition of $f \circ g, g \circ f$ and $\frac{f}{g}$.

2. Sketch the graph of

$$f(x) = \begin{cases} 3 - x^2 & x \leq -1 \\ 2 & -1 < x < 1 \\ 1 - x & x \geq 1 \end{cases}$$

and determine where f is discontinuous and justify your conclusion.

3. (a) Make a rough sketch of the graph of a function $f(x) = 2 + 3\ln(x-1)$.
(b) Find the domain and the range of the function.
(c) Find a formula for the inverse function f^{-1} of f and sketch the graph of $y = f^{-1}(x)$.

4. Solve the equations:
(a) $\log_2(\ln x) = 1$
(b) $2^{x-1} = 3^{x+1}$

5. Radium has a half-life of 1600 years. How many years does it take for 90% of a given amount to decay?

6.

(a) $\lim_{x \to 1^-} f(x)$
(b) $\lim_{x \to 1^+} f(x)$
(c) $\lim_{x \to 1} f(x)$
(d) $\lim_{x \to 3} f(x)$
(e) At which values of x is f not continuous?
(f) At which values of x is f not differentiable?

7. Find the limits if they exist.
   (a) $\lim_{x \to 5^-} \dfrac{x^2 - 3x - 10}{x^2 - 25}$
   (b) $\lim_{x \to 9} \dfrac{3 - \sqrt{x}}{9 - x}$
   (c) $\lim_{x \to -\infty} \dfrac{3 - x^3}{x^2 + 5x + 5}$

8. Find all vertical and horizontal asymptotes of the function $f(x) = \dfrac{2x - 3}{x + 2}$.

9. State the definition of the derivative in terms of limits. Use this definition to find the derivative of $f(x) = \dfrac{1}{1 - x}$.

10. If $y = f(x)$ satisfies $f(2) = 5$ and $f'(2) = 3$, find the equation of the tangent line to $y = f(x)$ at $x = 2$.

11. Evaluate the following derivatives:
    (a) $\dfrac{d}{du}(u^2 + 3u - \dfrac{2}{u})^4$
    (b) $\dfrac{d}{dx}(\sqrt{\sin \pi x} \cdot \cos \pi x)$
    (c) $\dfrac{d}{dt} \dfrac{\sin t}{t^2 - 5}$
    (d) $\dfrac{d}{dt} \tan(t^2 + 1)$
    (e) $\dfrac{d}{dx} \sqrt{e^{2x} + 1}$

12. Suppose that $h(x) = f(x)g(x)$ and $F(x) = f(g(x))$, where $f(2) = 3, g(2) = 5, g'(2) = 4, f'(2) = -2$, and $f'(5) = 11$. Find (a) $h'(2)$ and (b) $F'(2)$.

13. At what points on the curve $y = \sin^2 x$ is the tangent horizontal?

14. Make a careful sketch of $f(x) = \sin x$ and below it make a rough sketch of the graph of an antiderivative F, given that $F(0) = 0$.

15. Let $f(x) = x^3 - 6x^2 + 9x - 5$.
(a) At what values of x does f have a local maximum or minimum?
(b) On what intervals is f increasing or decreasing?
(c) On what intervals is the graph of f concave up or down?
(d) Make a rough sketch of the graph of f.

16. The Census figures for the US population (in millions) are listed in the table below. Let f be the function such that $P = f(t)$ is the population (in millions) in year t.

t	1950	1960	1970	1980	1990
$f(t)$	150.7	179.0	205.0	226.5	248.7

(a) Estimate the rate of change of the population for the year 1960.
(b) What does the derivative of $f(t)$ at $t = 1960$ represent?

17. The following graphs are the graph of a function $y = f(x)$ and its first derivative $y = f'(x)$.

Which graph is which? Give reasons for your answer.

18. Suppose that the cost, in dollars, for a company to produce x pairs of a new line of jeans is
$C(x) = 1000 + 2x + 0.1x^2 + 0.01x^3$
(a) Find $C'(x)$, the marginal cost function.
(b) Find $C'(10)$ and explain the meaning.

19. In December 2001, a Swedish pharmaceutical company released one of its best-selling anti-allergic drugs, X, in U.S. market. The company's monthly revenue $R(t)$ (in millions of dollars) from selling X in U.S. is approximately given by the formula: $R(t) = 25(1 - e^{-0.2t})$ where t represents the number of months since December 2001.

(a) Compute $R'(t)$. Is $R'(t)$ an increasing or decreasing function?

(b) Suppose this model will continue to hold for the future, what do you expect to see about the sales of X in U.S. in the long run? Explain your answer.