



# Math 214-1 Midterm

Fall Quarter 2002

Wednesday, October 30, 2002

Check your instructor's name and section:

Myung 8:00		Bode 11:00	
Gasper 9:00		Wahl 12:00	
Liang 9:00		Wahl 1:00	
Bode 10:00		Song 1:00	

Prob.	Possible points	Score
1	12	
2	6	
3	4	
4	8	
5	8	
6	6	
7	12	
8	8	
9	10	
10	16	
11	10	
TOTAL	100	

**Instructions:**

Show *all* your work on these sheets. Feel free to use the opposite side. Make sure that your final answer is clearly indicated. No calculators, books, notes, etc. are allowed. Good luck!

1. (12 points) Let  $f(x) = x^2 - 1$  and  $g(x) = \sqrt{x - 3}$ .

(a) Find the domain for each of the functions  $f \circ g$  and  $g \circ f$  .

(b) Find a formula for the inverse function  $g^{-1}(x)$  and sketch its graph.

2. (6 points) Solve the equations:

(a)  $\ln(3x - 1) = 0$

(b)  $2^{x+1} = 5$

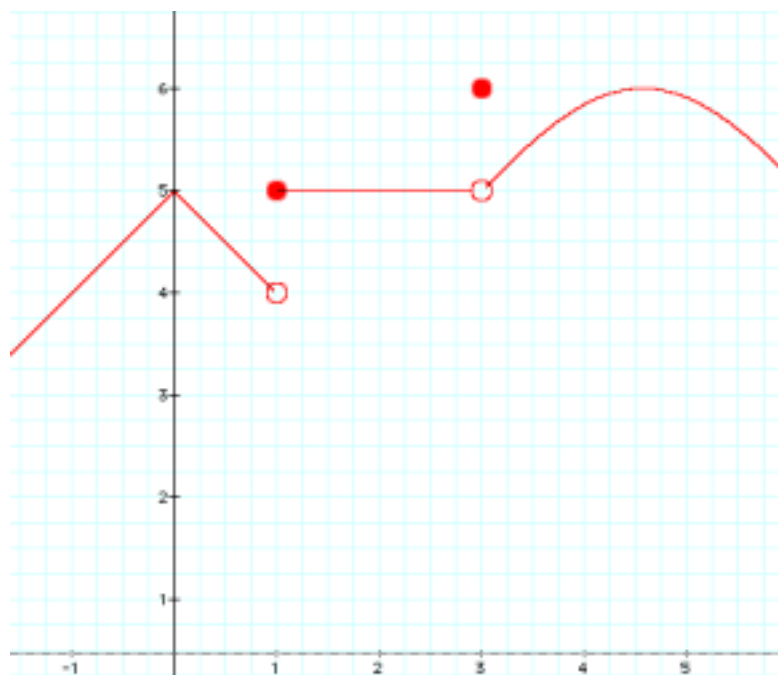
3. (4 points) Calculate:

(a)  $\log_{27} 3$

(b)  $\ln e^2$

4. (8 points) On what intervals is the function  $f(x) = xe^x$  increasing?
5. (8 points) Find an equation of the tangent line to the curve  $y = 2 + \sqrt{x}$  at the point  $(1, 3)$ .

6. (6 points) The graph of the function  $y = f(x)$  is given below.



(a) Determine whether the following limits exist, and evaluate those which exist.

$$\lim_{x \rightarrow 1} f(x)$$

$$\lim_{x \rightarrow 2} f(x)$$

(b) At which value(s) of  $x$  is  $f$  continuous, but not differentiable?

7. (12 points) Evaluate the following limits:

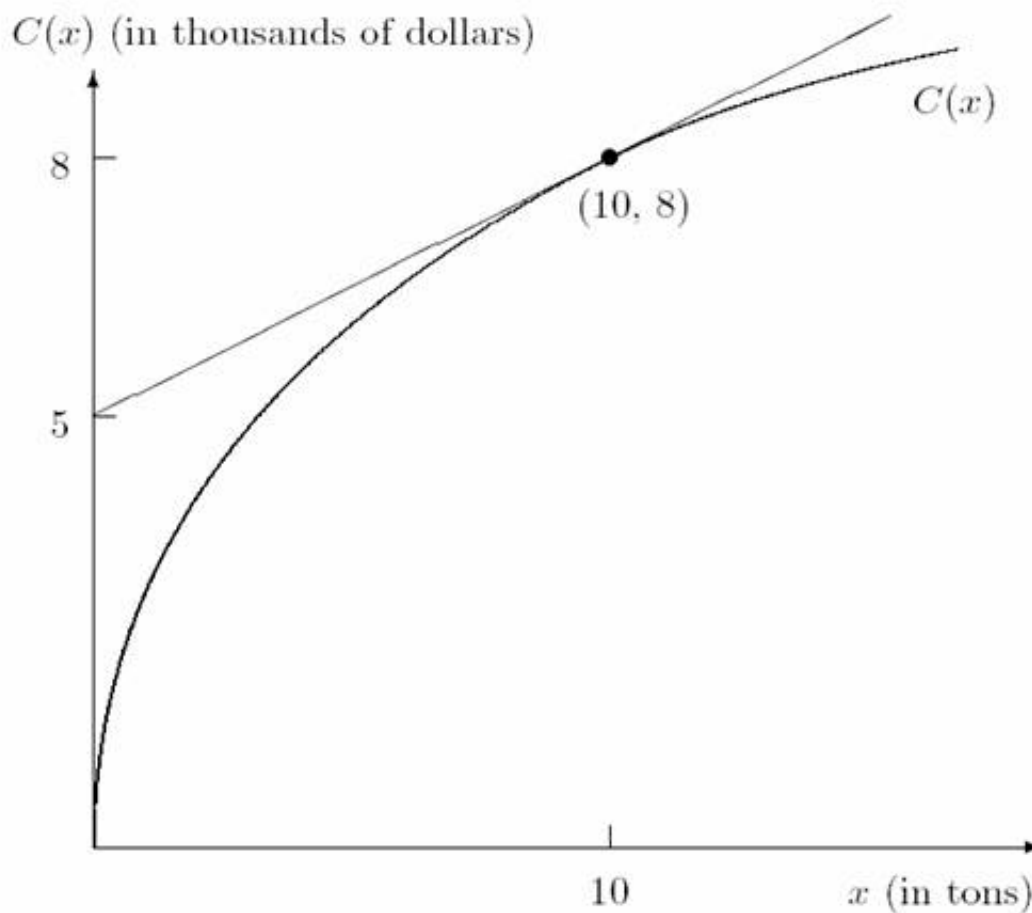
(a)  $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 5x + 6}$

(b)  $\lim_{x \rightarrow 2^+} \frac{2x^2 - 3}{x - 2}$

(c)  $\lim_{x \rightarrow \infty} \frac{2x^2 - x + 1}{3x^2 - 2}$

8. (8 points) Use the definition of the derivatives in terms of limits to find the derivative of  $f(x) = \sqrt{x+2}$  .

9. (10 points) Suppose  $C(x)$  (in thousands of dollars) represents the cost of manufacturing  $x$  tons of a chemical. The graphs of  $C(x)$  and its tangent line at  $(10, 8)$  are shown below. Note that the graph is **NOT** drawn to scale.



- (a) Compute  $C'(10)$ . Give units with your answer. Find the equation of the tangent line to  $C(x)$  at  $(10, 8)$ . Show all your work.
- (b) What is the meaning of the derivative  $C'(10)$ ?

10. (16 points) Differentiate the following functions. You do not need to simplify your answers.

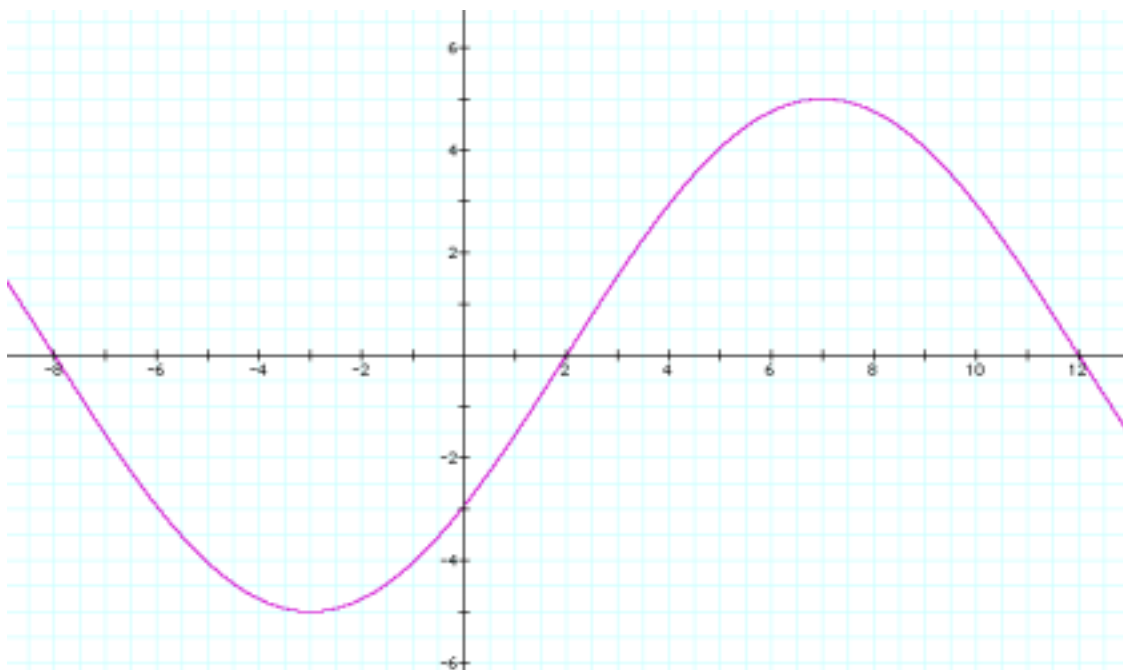
(a)  $f(x) = x^5 - \frac{1}{\sqrt{x^3}}$

(b)  $g(t) = \frac{\cos t}{2 \sin t + 3}$

(c)  $h(u) = \cos(e^{2u})$

(d)  $k(x) = e^{3x} \tan x$

11. (10 points) The graph of the **derivative**  $f'$  of a function  $f$  is given for  $-9 \leq x \leq 13$  by:



- (a) At what values of  $x$  does  $f$  have a local maximum or minimum?
- (b) On what intervals is  $f$  increasing or decreasing?
- (c) On what intervals is the graph of  $f$  concave up or down?