Mathematics 210-1, SAMPLE FINAL EXAMINATION, Fall 2002

1. Give definitions of:
(a) disjoint sets;
(b) independent events;
(c) the median;
(d) equivalent systems of linear equations;
(e) a dominated strategy for player B.

2. Construct a truth table for the following statement, then simplify the statement. Draw the circuit representing the simplified statement.

$$\sim (p \vee q) \to (\sim p \wedge \sim q)$$

3. Let U be the universal set, and let A and B be its subsets. If $n(U) = 60$, $n(A) = 28$, $n(A \cap B) = 8$, and $n(B') = 32$, then what is $n(A \cup B)$?

4. If the odds in favor of a baseball team winning their next game are 7 to 5, what is the probability that the team will win the game?

5. In a "quality of life" survey, 80% of the respondents lived in urban communities and 20% lived in rural areas. 70% of the urban residents said they were satisfied with their quality of life, while 98% of the rural folks said so. Suppose that a given respondemt indicated satisfaction with the quality of life. Find the probability that the individual lived in a rural area. Write the answer as a fraction.

6. A child has a set of differently shaped plastic objects. There are 3 pyramids, 2 cubes and 4 spheres. In how many ways can she arrange the objects in a row if each is a different color and objects of the same shape must be grouped together?

7. Three cards are drawn from an ordinary 52-card deck. In how many ways is it possible to draw all face cards?

8. A flat of petunias contains 5 seedlings with white flowers, 7 seedlings with red flowers, and 6 seedlings with purple flowers. Seven plants are selected at random and planted. Find the probability that there will be 3 white and 4 purple flowers among them.

9. A raffle has a first prize of \$ 300, two second prizes of \$ 100 each, and five third prizes of \$ 20 each. One thousand tickets are sold at \$ 1 a piece. Find the expected winnings of a person buying 1 ticket. Write the answer as a desimal.

10. A machine produces items having a mean length of 15.8 centimeters, with a standard deviation of .6 centimeters. The lengths are normally distributed. Find the probability that an item will have a length less than 15 centimeters.

11. Find the mean and the standard deviation for the following data.

Value	Frequency
2	4
4	8
6	10
8	5
10	3

12. Solve the following system. Check your answer. If a system has infinitely many solutions, use x as a parameter.

$2x - 3y = 4$
$4x - 6y = 8$

13. Use the Gauss-Jordan method to solve the following system of linear equations. Check your answer.

$2x - y + 2z = 10$
$x + 2y + 3z = 10$
$3x - y - z = 2$

14. Let $A = \begin{pmatrix} -3 & -2 \\ 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 0 & -1 \\ -3 & 2 & 6 \end{pmatrix}$. Find the products AB and BA, if these products exist.

15. Use the graphical method to solve the following linear programming problem.

Maximize $z = 3x + 5y$
subject to: $y - x \leq 2$
$x + y \leq 4$
$x \geq 0$
$y \geq 0.$

16. POPS International makes two kinds of tables: cocktail tables and end tables. It takes 2 hours to assemble and 3 hours to apply the finish to each cocktail table for a profit of $ 12. It takes 4 hours to assemble and 1 hour to apply the finish to each end table for a profit of $ 10. If the assembly operation is limited to 20 hours a day and the finishing step is limited to 15 hours a day, how many tables of each type should the company make to maximize profit? Set up a system of inequalities for this problem, identify all variables used, and give the objective function, but do not solve.

17. Remove any dominated strategies if they exist. Then check for a saddle point. Identify any strictly determined games. Find the optimum strategies for players A and B. Give the value of the game. Is the game fair?

(a) $\begin{pmatrix} -3 & -5 & 8 \\ 2 & 4 & 6 \\ 1 & 5 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & -2 & 2 & 8 \\ -1 & 0 & 3 & 1 \\ -2 & -5 & 0 & -3 \end{pmatrix}$