

Final Exam
(2 hours, 200 points)

1. (25pts) Suppose E and F are two events such that $P(E) = .3$, $P(F) = .5$, and $P(E \cup F) = .7$. Find $P(E \cap F)$ **and** $P(E' \cup F')$.
2. (25pts) How many ways can 4 boys and 3 girls be seated in a row if boys must sit together? How many ways are there if no boy is to sit next to another boy?
3. (25pts) A hand of 5 cards is dealt from a standard deck of 52-cards. Find the probability the hand consists of
 - a) (15pts) 3 spades and 2 hearts.
 - b) (10pts) 3 of one suit and 2 of another suit.
4. (25pts) A pair of dice is rolled. Let A be the event that the sum is 7 and B the event that one die shows a four. Compute
 - a) (5pts) $P(A|B)$
 - b) (10pts) Are A and B mutually exclusive? Explain your reasoning.
 - c) (10pts) Are A and B independent? Explain your reasoning.
5. (25pts) A building contractor buys 70% of his cement from supplier A and 30% from supplier B . A total of 90% of the bags from A arrive undamaged, while 95% of the bags from B arrive undamaged. Find the probability that a damaged bag came from supplier A .
6. (25pts) Heavy-duty resistors have an expected lifetime of 5 years with a variance of 0.16. If 3000 such resistors are installed, estimate how many will last between 3 and 7 years? (Hint: Use Chebyshev's Theorem.)
7. (25pts) A biased coin is tossed 100 times. Suppose the probability of obtaining heads on each toss is .4. Let X be the random variable representing the number of heads observed.
 - a) (5pts) Compute μ and σ for the random variable X .
 - b) (10pts) Find an exact expression for $P(X \geq 3)$.
 - c) (10pts) Use normal approximation to estimate $P(X \geq 3)$.
8. (25pts) Consider a betting game where a die is rolled and you win/lose X dollars if an even/odd number shows, where X is the number rolled.
 - a) (7pts) What is the probability you lose \$9 or more after two games?
 - b) (8pts) What is the probability you win \$9 or more after two games?
 - c) (10pts) What is the expected winnings per game?
 - d) (+5pts) After playing 100 games you find yourself 10 dollars short. You suspect the die is loaded and compute χ^2 using the data from the 100 games. Suppose you find that χ^2 is less than the 5% critical level. What can you conclude from this? What is the likelihood that your conclusion is correct?