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Problem A1. Prove that the following equation has no solutions in positive integers:

$$8x^4 + 4y^4 + 2z^4 = t^4 .$$

(Hint: t must be an even integer.)

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Problem A2. Let $a_1, a_2, a_3, \dots$ a strictly increasing sequence of positive integers, i.e., $a_n \in \mathbb{Z}^+ = \{1, 2, 3, \dots\}$, and $n < m \Rightarrow a_n < a_m$ for every m, n. Find all strictly increasing functions functions $f : \mathbb{Z}^+ \to \mathbb{Z}^+$, where $\mathbb{Z}^+ = \{1, 2, 3, \dots\}$, such that $f(a_n) \leq a_n$ for every $n \in \mathbb{Z}^+$.

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Problem A3. Find the following limit:

$$L = \lim_{n\to\infty} \sqrt[n]{\prod_{k=1}^{n} \left(1 + \frac{k}{n}\right)^{1/(1+\frac{k}{n})}} .$$

(Hint: Take the logarithm of the expression under the limit.)

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Problem A4. A fair coin is tossed repeatedly. What is the expected number of times the coin will be tossed until getting two heads in a row for the first time?

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Problem A5. Let a_k, $k = 1, 2, 3, \ldots$, be a sequence of strictly positive numbers of period $2N$. Show that

$$\sum_{j=1}^{2N} \frac{a_{N+j}}{a_j} \geq 2N\,.$$

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Problem A6. Given any positive integer a consider the sequence $a_n = a^{a^{a^n}}$, $n = 1, 2, 3, \ldots$. Prove that regardless of the integer a chosen, the rightmost digit of the decimal representation of a_n remains constant.