Geometry and Topology preliminary exam, Fall 2018

All numbered questions are weighted equally, and all lettered questions are weighted equally within their number. You may cite any result which was lectured on in class, as much as possible state any cited theorem clearly.

► 5: Let $f : \mathbb{CP}^m \to \mathbb{CP}^n$ be any continuous map between complex projective spaces of dimension m and n.

▷ 5a: If $m > n$, show that the induced map $f_* : H_k(\mathbb{CP}^m, \mathbb{Z}) \to H_k(\mathbb{CP}^n, \mathbb{Z})$ is zero for all $k > 0$.

▷ 5b: If $m = n$, the induced map $f_* : H_{2m}(\mathbb{CP}^m, \mathbb{Z}) \cong \mathbb{Z} \to H_{2m}(\mathbb{CP}^n, \mathbb{Z}) \cong \mathbb{Z}$ is given by multiplication by some integer d, called the degree of f. Find all possible values of d for the continuous map f.

► 6: Let X be a CW-complex with a single cell in each of dimensions $0, 1, 2, 3, 5$ and no other cells.

▷ a: What are the possible cohomology groups of $H^*(X, \mathbb{Z})$? (note it is not sufficient to consider $H^n(X, \mathbb{Z})$ for each n independently. The values of $H^1(X, \mathbb{Z})$ might affect $H^2(X, \mathbb{Z})$, for instance.)

▷ b: What happens to part (a) if X is simply connected?